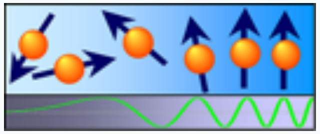
Experimental Physics EP2a

Electricity and Wave Optics

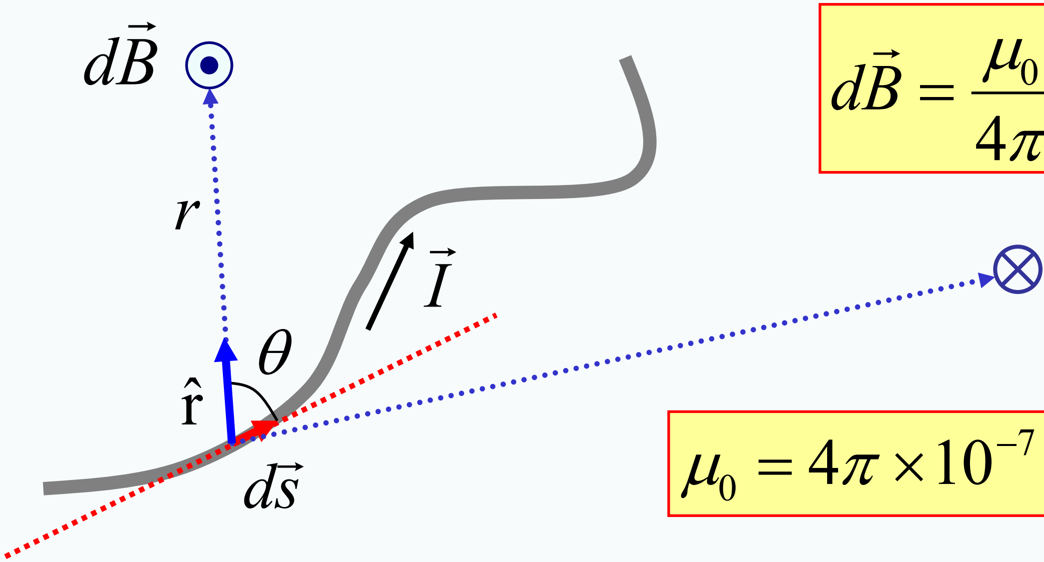
– Sources of magnetic field –

Current elements, Ampere's law



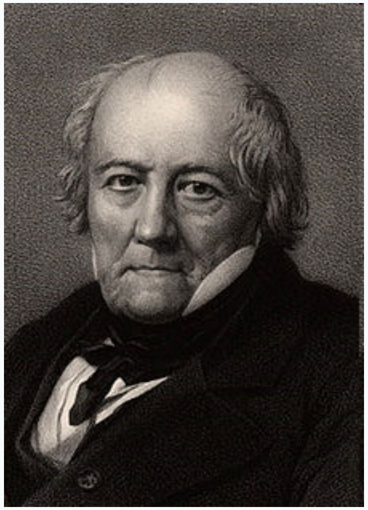
https://bloch.physgeo.uni-leipzig.de/amr/

The Biot-Savart law



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$



Jean-Baptiste Biot

Félix Savart

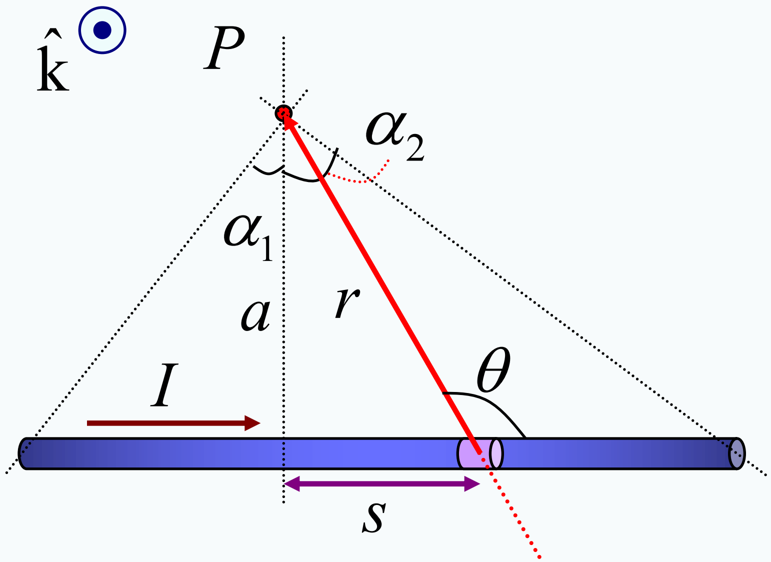
The vector dB is perpendicular both to ds and to the vector directed from ds to the point of interest P.

The magnitude of dB is inversely proportional to r^2, where r is the distance from ds to P.

The magnitude of dB is proportional to the current and to the length of ds.

The magnitude of dB is proportional to sin(θ), where θ is the angle between the vectors ds and r.

Magnetic field of a rod



$$d\vec{B} = dB \cdot \hat{k}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

$$\frac{d\vec{s} \times \hat{r}}{r^2} = \frac{ds \cdot \sin\theta}{r^2} \hat{k}$$

$$s = -r\cos\theta \qquad r = \sqrt{a^2 + s^2}$$

$$ds = \frac{r d\theta}{\sin\theta} \qquad \frac{ds \cdot \sin\theta}{r^2} = \frac{\sin\theta d\theta}{a}$$

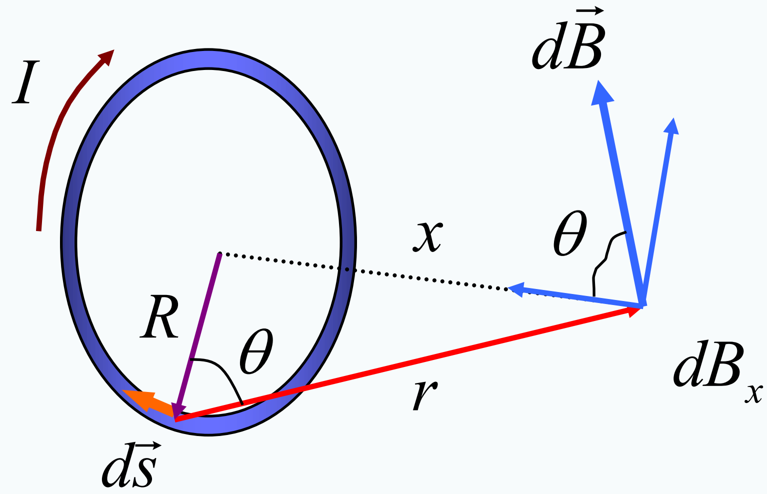
$$\vec{B} = \frac{\mu_0 I}{4\pi a}\left(\sin\alpha_1 + \sin\alpha_2\right)\hat{k}$$

$$\int_{\pi/2-\alpha_1}^{\pi/2+\alpha_2} \sin\theta d\theta = \sin\alpha_2 + \sin\alpha_1$$

Infinite straight wire:

$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{k}$$

Magnetic field of a ring



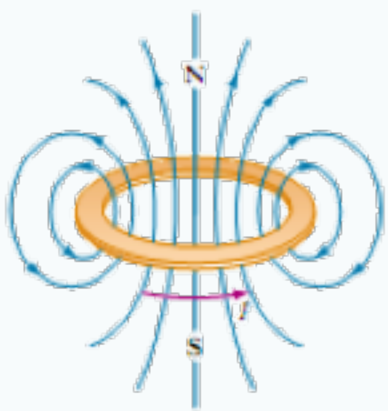
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{\mathrm{r}}}{r^2}$$

$$\int dB_y = 0$$

$$\left| d\vec{s} \times \hat{\mathrm{r}} \right| = ds$$

$$dB_x = dB \cos\theta$$

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{\cos\theta ds}{\left(x^2 + R^2\right)} = \frac{\mu_0 I}{4\pi}\frac{R ds}{\left(x^2 + R^2\right)^{3/2}}$$



$$B_x = \frac{\mu_0 I}{2}\frac{R^2}{\left(x^2 + R^2\right)^{3/2}}$$

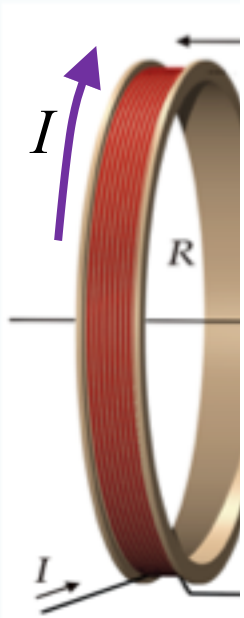
The ring center: $B_x = \frac{\mu_0 I}{2R}$

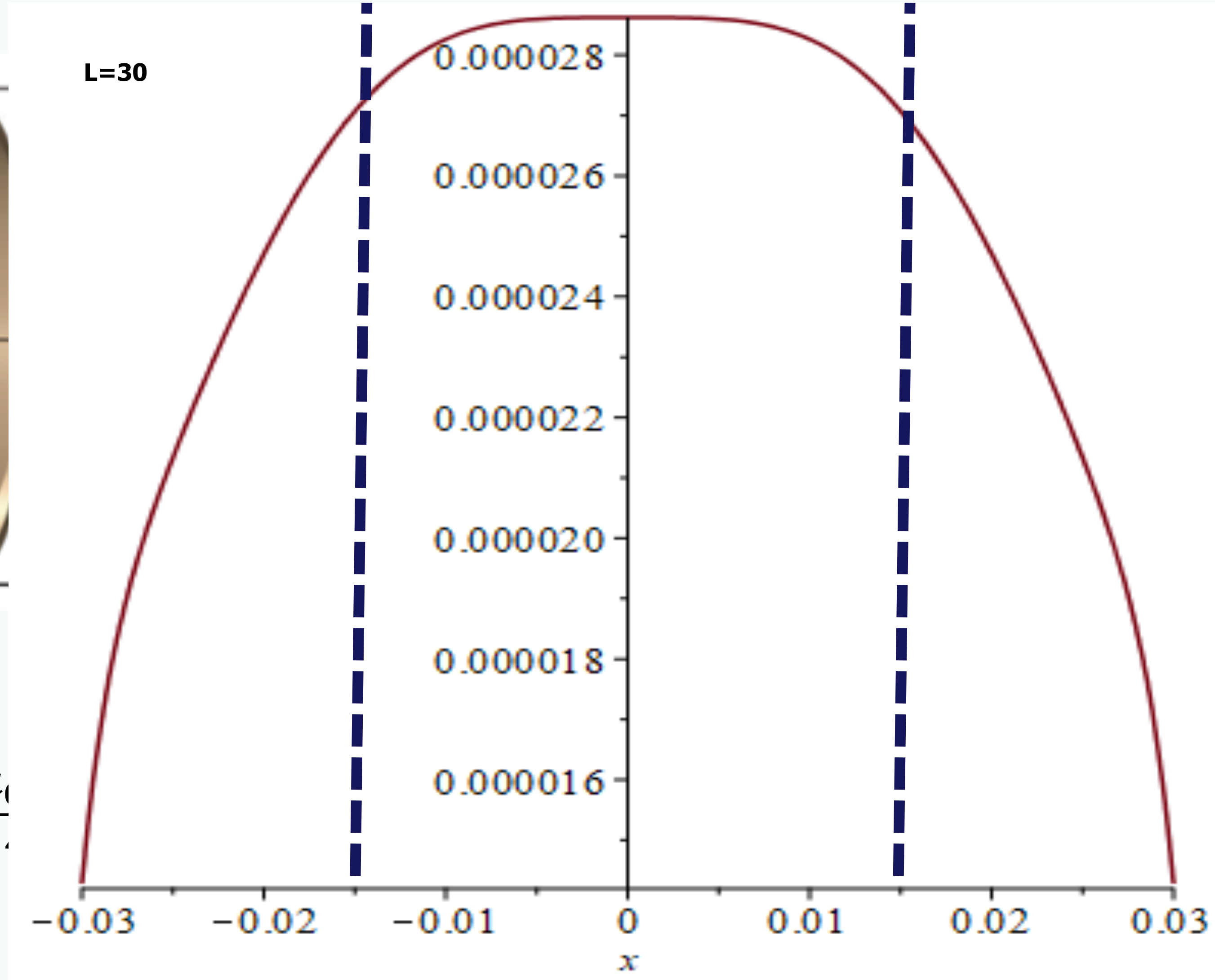
Far away along the symmetry axis:

$$E_x = \frac{kxQ}{\left(x^2 + R^2\right)^{3/2}}$$

$$B_x = \frac{\mu_0 I}{2}\frac{R^2}{x^3} = \frac{\mu_0}{2\pi}\frac{\mu}{x^3}$$

Helmholtz coil



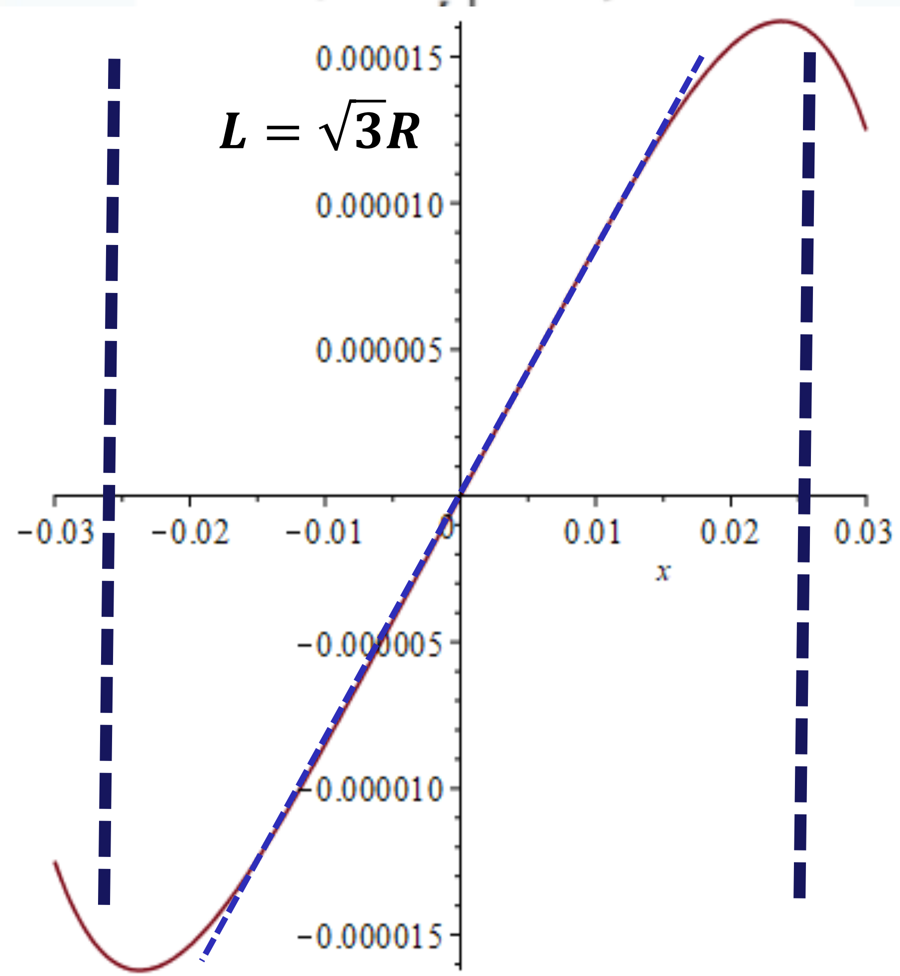


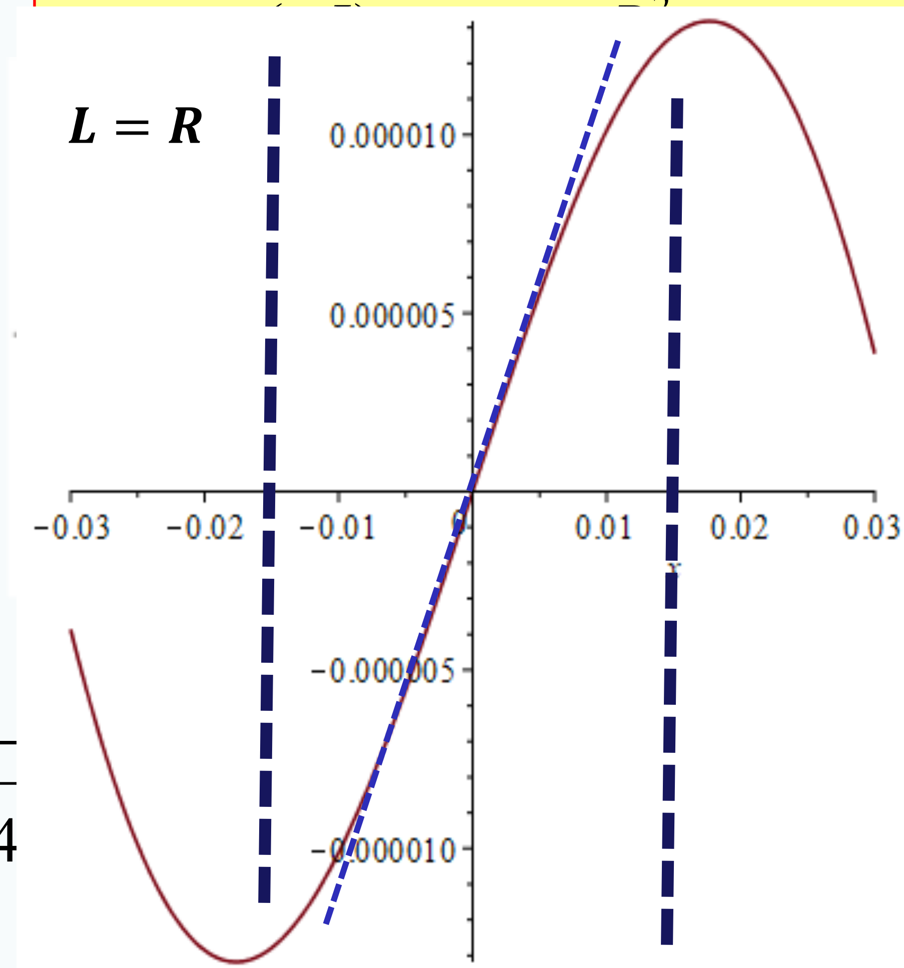
$\ldots R^2)^{3/2}$

$\ldots R^2)^{3/2}$

$B_x = \frac{8\mu_{\ldots}}{(L^2 + \ldots} \ldots (x^6)$

Maxwell pair

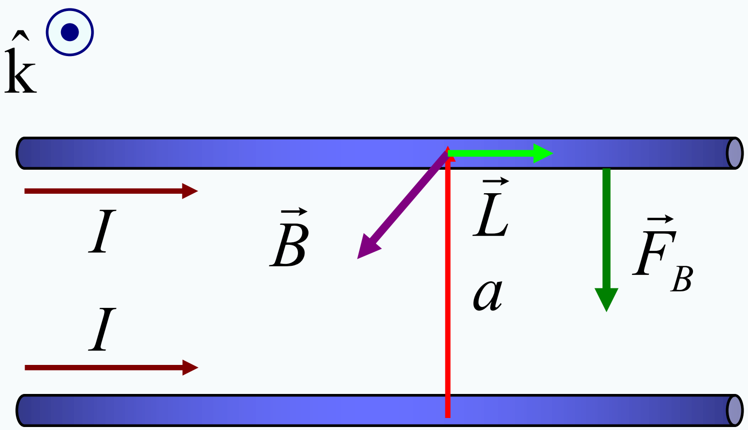




$L = \sqrt{3}R$

$$g_x \equiv \frac{dB_x}{dx} = \frac{48\sqrt{21}}{343}\frac{\mu_0 I}{R^2}$$

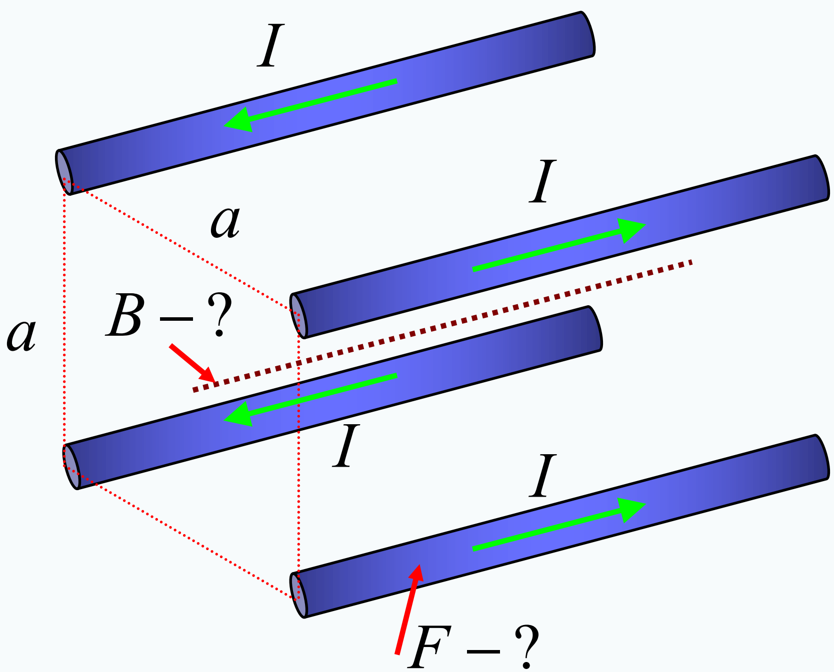
Force between two wires

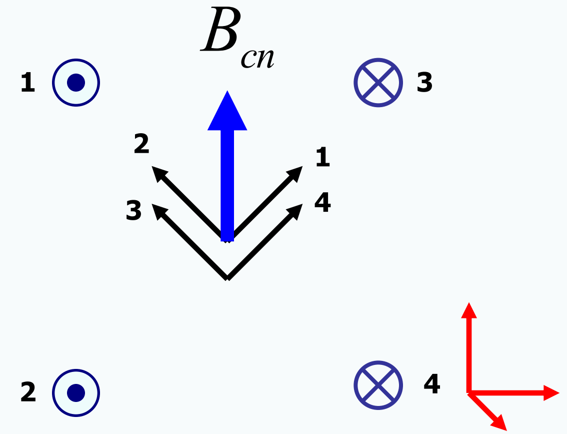


$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{\mathrm{k}}$$

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

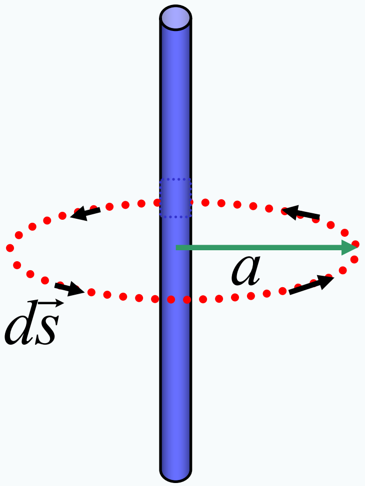
$$\left|\vec{F}_B\right|_{\uparrow\uparrow} = \frac{\mu_0 I_1 I_2 L}{2\pi a} \qquad \left|\vec{F}_B\right|_{\uparrow\downarrow} = -\frac{\mu_0 I_1 I_2 L}{2\pi a}$$





$$B_{cn} = \frac{2\mu_0 I}{\pi a} \qquad F_{net,4} = \frac{\sqrt{10}}{2}\frac{\mu_0 I^2 L}{2\pi a}$$

Ampere's law



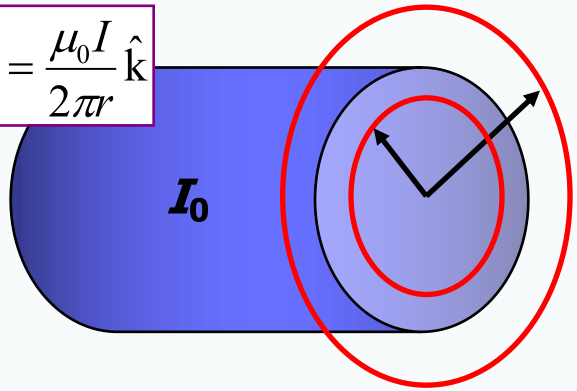
$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{\mathrm{k}}$$

$$\oint \vec{B}\cdot d\vec{s} = \oint B ds$$

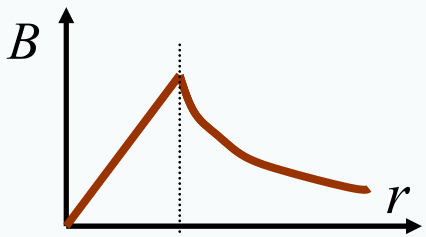
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

The line integral of B ds around any closed path equals $\mu_0 I$, where I is the total continuous current passing through any surface bounded by the closed path.

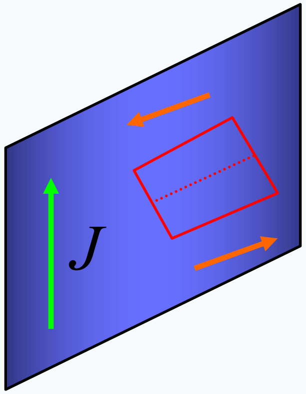
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\mathrm{k}}$$



$$B_{in} = \frac{\mu_0 I_0}{2\pi R^2} r$$



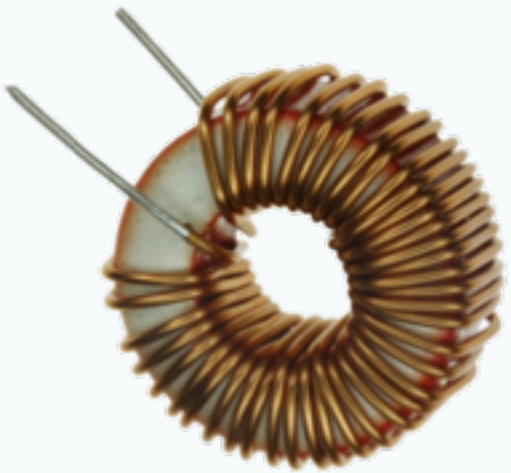
$$B_{in}\cdot 2\pi r = \mu_0 nqv_d \pi r^2 = \mu_0 I_0 \frac{r^2}{R^2}$$



$$2BL = \mu_0 JL$$

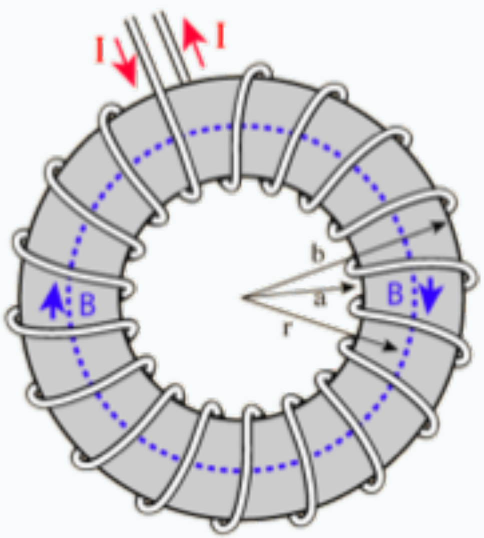
$$B = \tfrac{1}{2}\mu_0 J$$

Magnetic field strength



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 N I \qquad \boldsymbol{B} = \frac{\boldsymbol{N}}{\boldsymbol{2\pi r}} \mu_0 I$$



$$B_x = \frac{\mu_0 I}{2} \frac{R^2}{\left(x^2 + R^2\right)^{3/2}}$$

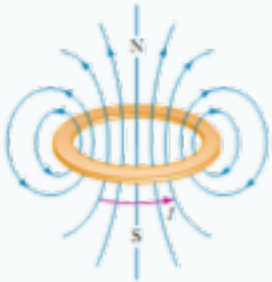
magnetic field due to a circular current

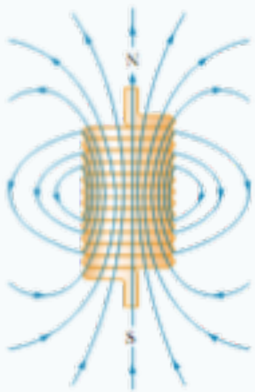
$$2\int_0^\infty B_x dx = \mu_0 I R^2 \int_0^\infty \frac{dx}{\left(x^2 + R^2\right)^{3/2}} = \mu_0 I \int_0^\infty \frac{dy}{\left(1 + y^2\right)^{3/2}}$$

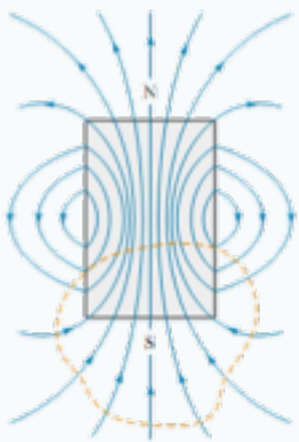
$$B_{toroid} = n \cdot \mu_0 I$$

n – the density of turns per unit length

Gauss's law of magnetism







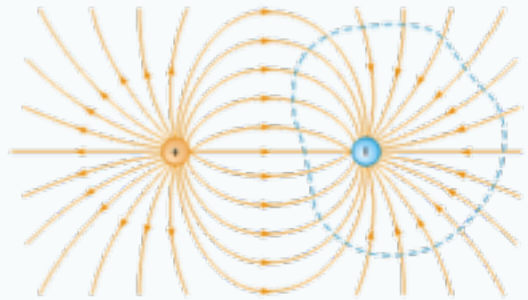
$$\Phi_B = \oint\!\!\oint \vec{B} \cdot d\vec{A}$$

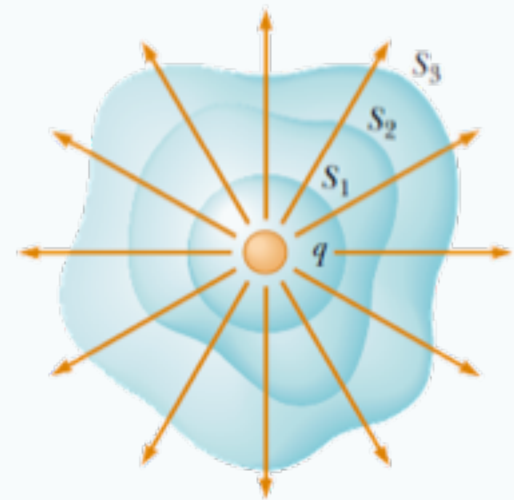
$$[\Phi_B] = \mathrm{Tm}^2$$ **Weber**

$$\oint\!\!\oint \vec{B} \cdot d\vec{A} = 0$$

The net magnetic flux through any closed surface is zero.

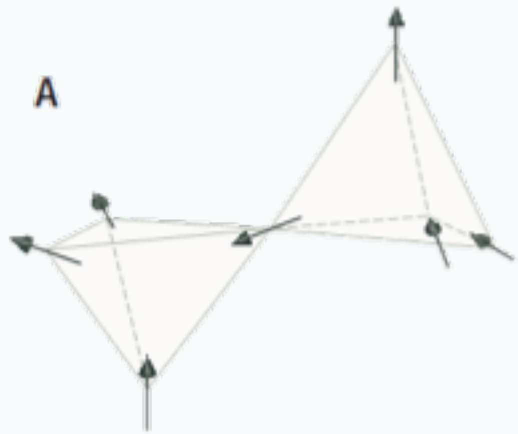
Isolated magnetic monopoles have never been detected.





$$\Phi_E = \oint\!\!\oint \vec{E} \cdot d\vec{A} = 4\pi k q_{in}$$

Magnetic monopoles



Dirac Strings and Magnetic Monopoles in the Spin Ice $Dy_2Ti_2O_7$

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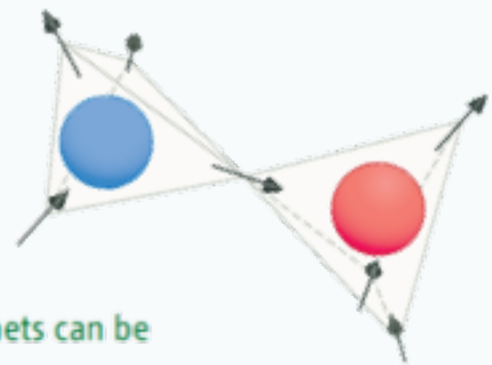
Sources of magnetic fields—magnetic monopoles—have so far proven elusive as elementary particles. Condensed-matter physicists have recently proposed several scenarios of emergent quasiparticles resembling monopoles. A particularly simple proposition pertains to spin ice on the highly frustrated pyrochlore lattice. The spin-ice state is argued to be well described by networks of aligned dipoles resembling solenoidal tubes—classical, and observable, versions of a Dirac string. Where these tubes end, the resulting defects look like magnetic monopoles. We demonstrated, by diffuse neutron scattering, the presence of such strings in the spin ice dysprosium titanate ($Dy_2Ti_2O_7$). This is achieved by applying a symmetry-breaking magnetic field with which we can manipulate the density and orientation of the strings. In turn, heat capacity is described by a gas of magnetic monopoles interacting via a magnetic Coulomb interaction.

PHYSICS

Observing Monopoles in a Magnetic Analog of Ice

Michel J. P. Gingras

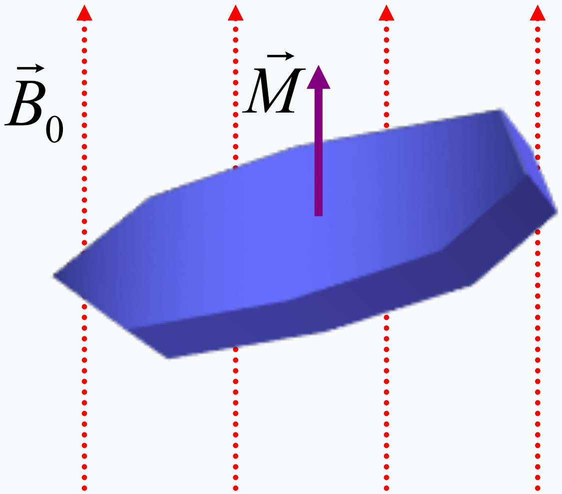
Experimental evidence has been found that magnetic poles within metal oxide magnets can be separated.



To remember!

- **The Biot-Savart law quantifies the magnetic field created by a length element currying an electric current.**
- **Magnetic field created by a straight wire decreases inversely proportional to the distance from the conductor.**
- **Two parallel wires currying electric currents attract each other if the currents are in the same direction and repel each other if the currents are anti-parallel.**
- **Ampere's law states that the line integral of B·ds around any closed path is proportional to the enclosed by the path electric current.**
- **Gauss's law of magnetism states that the net magnetic flux through any closed surface is zero.**

Magnetic field strength

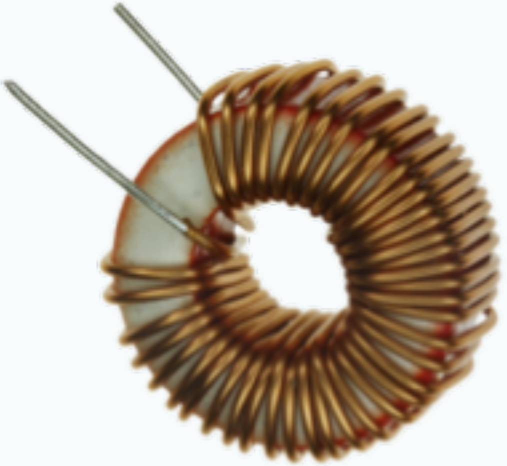


$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M} = \mu_0 \vec{H}_0 + \mu_0 \vec{M}$$

M is the magnetization vector
H is the magnetic field strength

$$B_x = \frac{\mu_0 I}{2} \frac{R^2}{\left(x^2 + R^2\right)^{3/2}}$$

magnetic field due to a circular current



$$2\int_0^\infty B_x dx = \mu_0 I R^2 \int_0^\infty \frac{dx}{\left(x^2 + R^2\right)^{3/2}} = \mu_0 I \int_0^\infty \frac{dy}{\left(1 + y^2\right)^{3/2}}$$

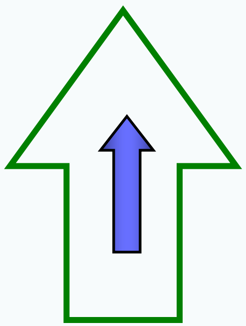
$$B_{toroid} = n \cdot \mu_0 I$$

$$H_{toroid} = nI$$

Magnetic substances

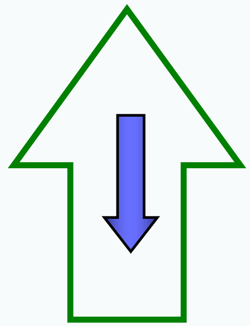
Paramagnetic

permanent



$$M = \chi H$$

Diamagnetic



Ferromagnetic

permanent

$$M = f(H)$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 (1 + \chi) \vec{H}$$

$$\vec{B} = \mu_m \vec{H}$$

Magnetic permeability

$$\text{Para, dia} \quad \mu_m \sim \mu_0$$

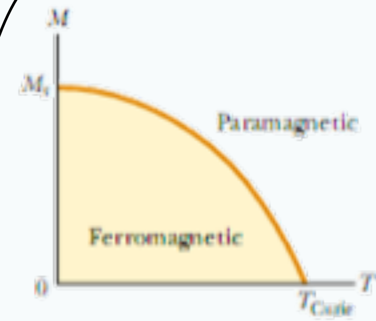
$$\text{Ferro} \quad \mu_m \sim \left(10^3 - 10^4\right) \mu_0$$

TABLE 30.2 Magnetic Susceptibilities of Some Paramagnetic and Diamagnetic Substances at 300 K

Paramagnetic Substance	χ	Diamagnetic Substance	χ
Aluminum	2.3×10^{-5}	Bismuth	-1.66×10^{-5}
Calcium	1.9×10^{-5}	Copper	-9.8×10^{-6}
Chromium	2.7×10^{-4}	Diamond	-2.2×10^{-5}
Lithium	2.1×10^{-5}	Gold	-3.6×10^{-5}
Magnesium	1.2×10^{-5}	Lead	-1.7×10^{-5}
Niobium	2.6×10^{-4}	Mercury	-2.9×10^{-5}
Oxygen	2.1×10^{-6}	Nitrogen	-5.0×10^{-9}
Platinum	2.9×10^{-4}	Silver	-2.6×10^{-5}
Tungsten	6.8×10^{-5}	Silicon	-4.2×10^{-6}

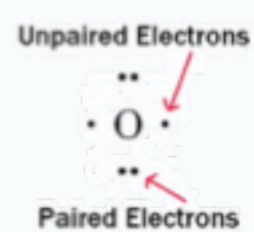
Magnetic substances

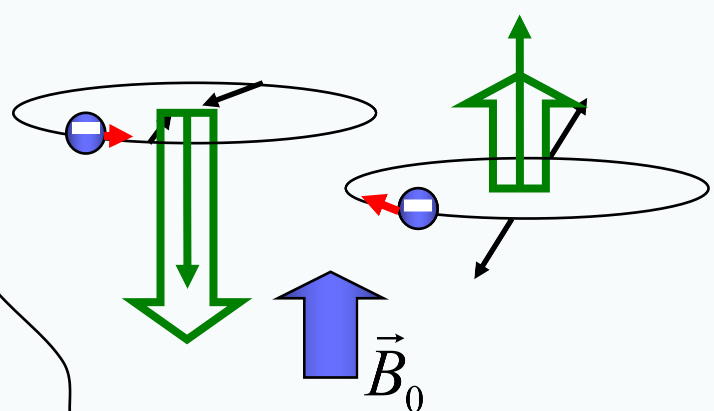
Ferromagnetism



$$M = C\frac{B}{T}$$

Paramagnetism



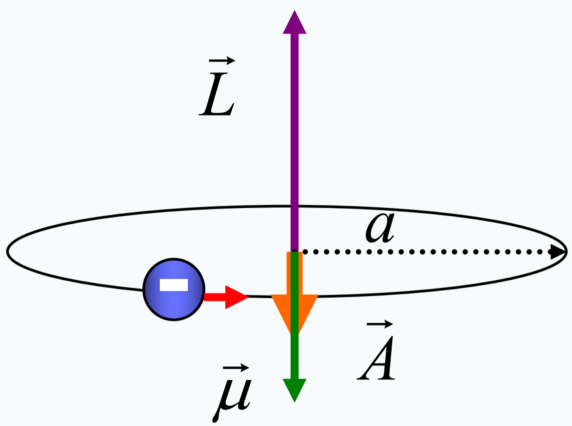


Diamagnetism

To remember!

- **Magnetic moments in magnetized substances arise due to atomic-level electric currents.**
- **Paramagnetic and ferromagnetic substances are those made up of atoms having permanent magnetic moments.**
- **In diamagnetic substances the atoms do not have permanent magnetic moment.**
- **Ferromagnetic substances contain small domains having the same alignments of the atomic magnetic moments.**
- **Ferromagnetic materials become paramagnetic above the Curie temperature.**

Magnetic moment of orbiting electron



$$\vec{\mu} = I\vec{A}$$

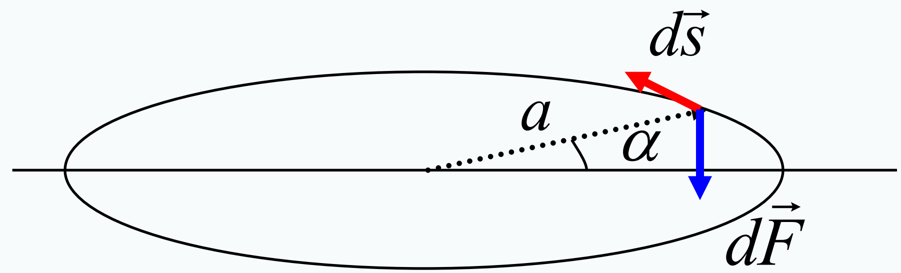
$$I = \frac{dq}{dt} = \frac{dq}{dl}\frac{dl}{dt} = \frac{ev}{2\pi a}$$

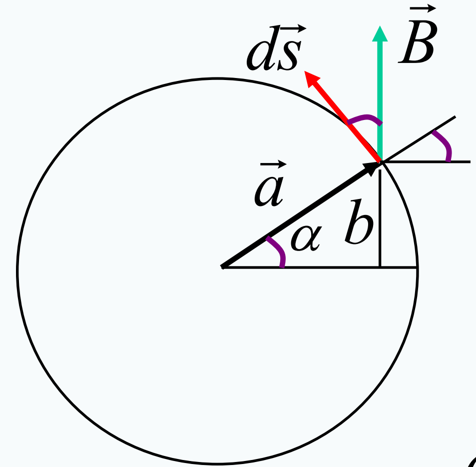
$$\mu = \tfrac{1}{2} eva \qquad \mu = \frac{e}{2m} L$$

$$L = I\omega = mav$$

$\odot\ \vec{B}$

$$dF = IB \sin \alpha ds$$





$$d\tau = bdF$$

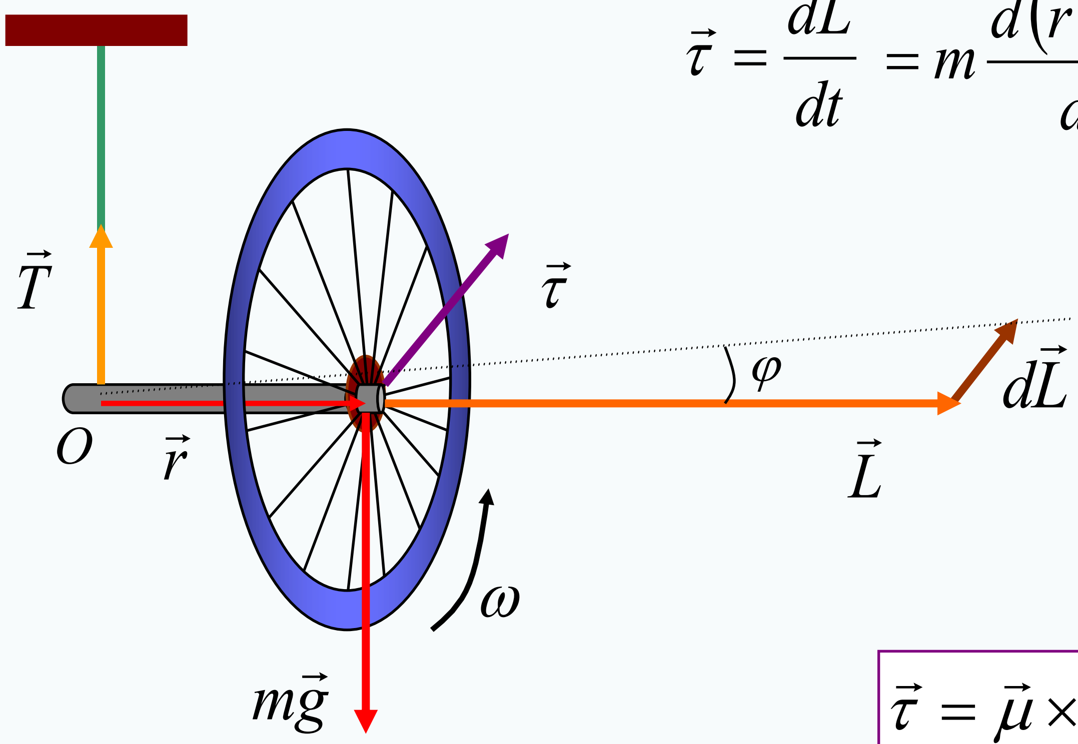
$$ds = ad\alpha$$

$$b = a \sin \alpha$$

$$d\tau = IBa^2 \sin^2 \alpha d\alpha$$

$$\tau = IBa^2 \int_0^{2\pi} \sin^2 \alpha d\alpha = IB\pi a^2$$

Motion of a wheel



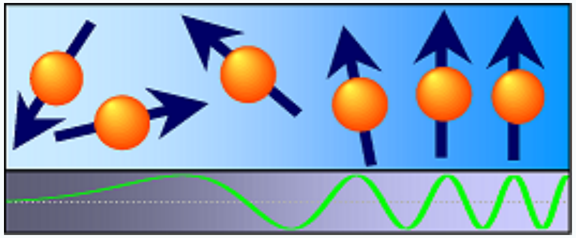
$$\vec{\tau} = \frac{d\vec{L}}{dt} = m\frac{d(\vec{r}\times\vec{v})}{dt} = m\vec{r}\times\frac{d\vec{v}}{dt} + m\frac{d\vec{r}}{dt}\times\vec{v}$$

$$dL = \tau dt = rmg dt = L d\varphi$$

$$\omega_p = \frac{d\varphi}{dt} = \frac{rmg}{L}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

$$\vec{\mu} = \frac{1}{2}\frac{Q}{M}\vec{L}$$



$$\omega = \frac{\tau}{L} = \frac{\mu B}{(2m/Q)\mu} = \frac{Q}{2m}B \equiv \gamma B$$