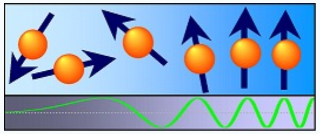
Experimental Physics EP2a

Thermodynamics & Electricity

– Magnetic field –
Isolated charges, conductors



https://bloch.physgeo.uni-leipzig.de/amr/

Some history

1269 **Pierre Pelerin de Maricourt**

1600 **William Gilbert**

1802 **Gian Domenico Romagnosi**

1819 **Hans Christian Ørsted**

1820, 11 September

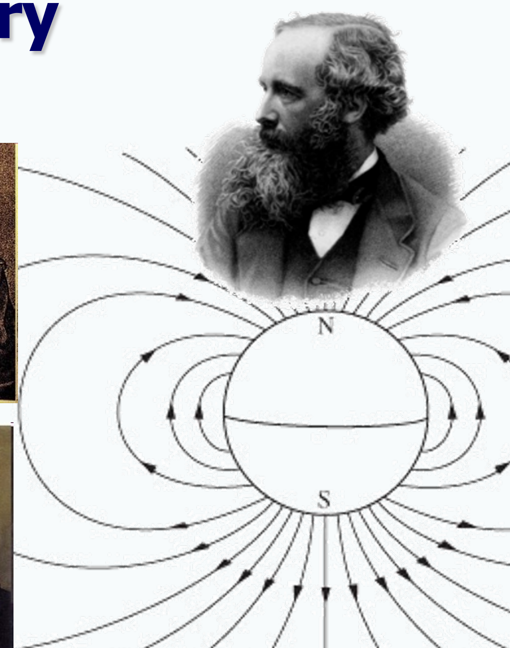
André-Marie Ampère

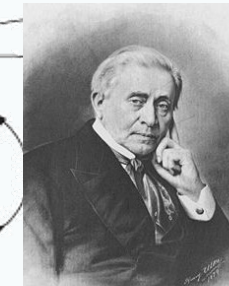
1820s **Michael Faraday**

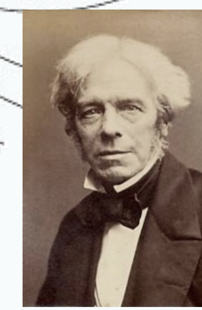
Joseph Henry

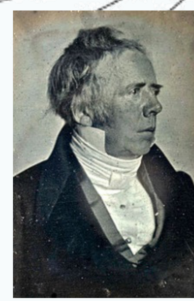
1865 **James Maxwell**

A Dynamical Theory of the Electromagnetic Field









Experimental observations

The magnitude of the magnetic force on a moving charge is proportional to its speed |v| and to its charge q.

The magnitude and the direction of this force depends on the magnitude |B| and the direction of the magnetic field.

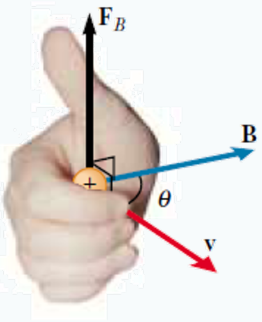
When a charged particle moves parallel to the magnetic field lines, the force F exerted upon the particle is zero.

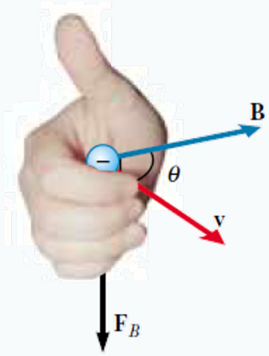
However, if the angle between the magnetic field and the particle velocity is not zero, the force is perpendicular to both v and B.

The magnitude of the magnetic forces will be proportional to sin(θ), where θ is the angle between the directions of v and B.

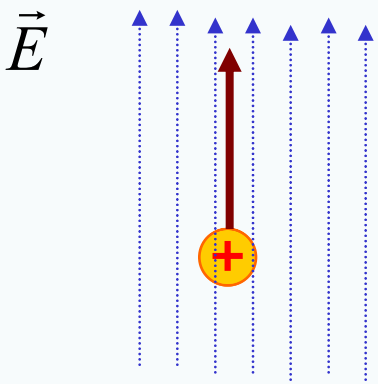
If two charged particles with opposite charges move parallel to each other, the forces exerted upon the particles will be in opposite directions.

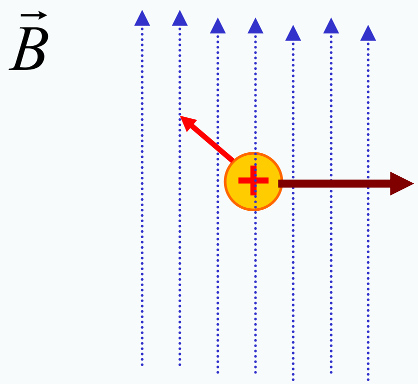
$$\vec{F}_B = q\vec{v} \times \vec{B}$$





How to probe magnetic field





$$W_E = \int \vec{F} d\vec{s}$$

$$W_B = 0 \qquad \vec{F} \perp \frac{d\vec{s}}{dt}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$1\,\mathrm{T} = 10^4\ \mathrm{G}$$

Magnetic field can alter only the direction of a charged particle moving in this field, but not the speed or kinetic energy of the particle.

$$[B] = \frac{\mathrm{N}}{\mathrm{C} \cdot \frac{\mathrm{m}}{\mathrm{s}}} = \frac{\mathrm{N}}{\mathrm{Am}} \equiv \mathrm{T}$$

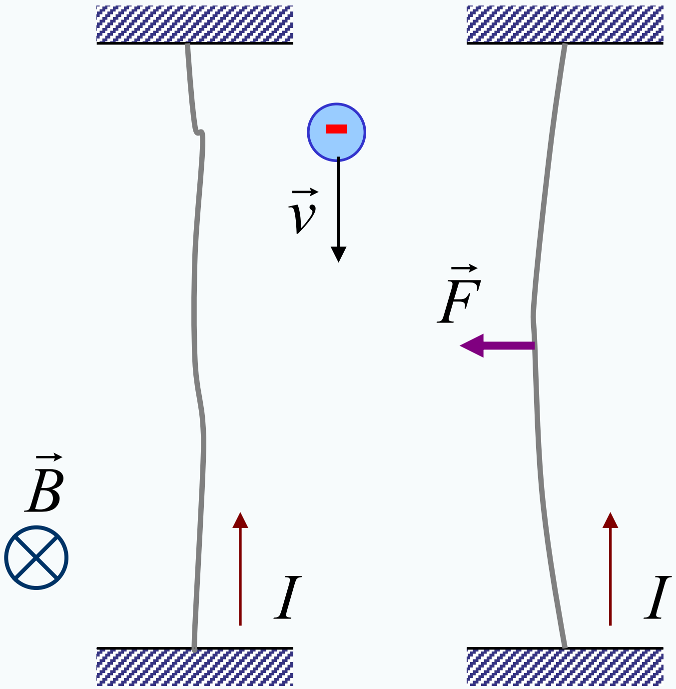
$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

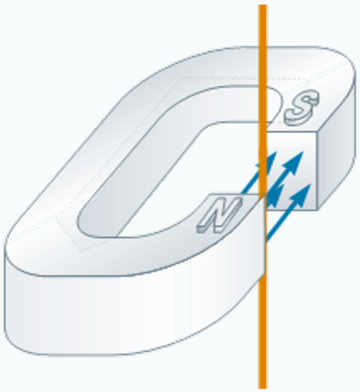
$$\vec{B} = \frac{\vec{F} \times \vec{v}_\perp}{q v_\perp^2}$$

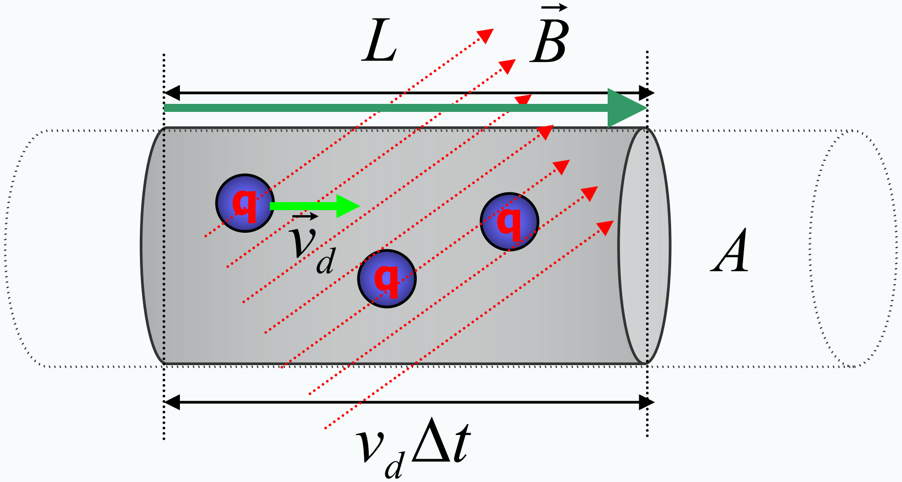
TABLE 29.1 Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

Conductors







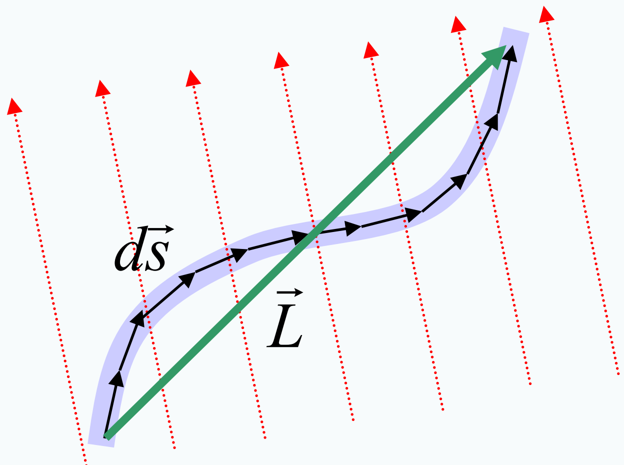
$$\vec{F}_B = Q\vec{v}_d \times \vec{B}$$

$$\vec{F}_B = nALq\left(\vec{v}_d \times \vec{B}\right)$$

$$I = \frac{Q}{\Delta t} = \frac{Q}{L} v_d = nAqv_d$$

$$\vec{F}_B = I\left(\vec{L} \times \vec{B}\right)$$

Arbitrarily shaped wires



$$d\vec{F}_B = I d\vec{s} \times \vec{B}$$

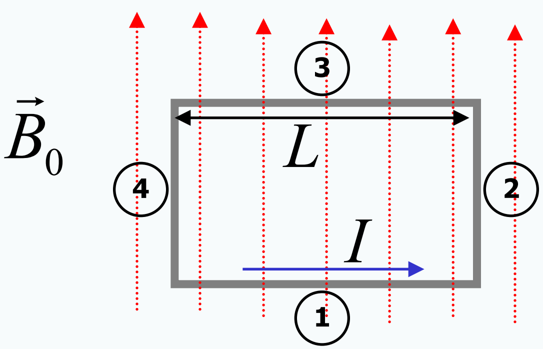
$$\vec{F}_B = I\left(\vec{L} \times \vec{B}\right)$$

$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B}$$

$$\vec{B} = \vec{B}(\vec{s})$$

$$\vec{B} = \vec{B}_0$$

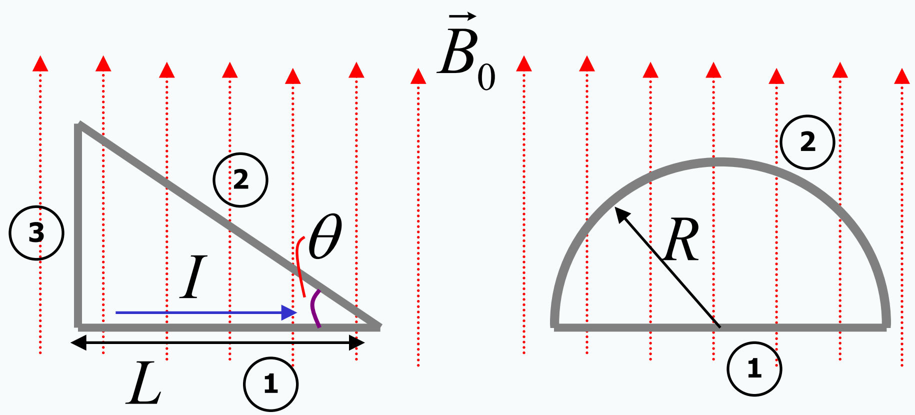
$$\vec{F}_B = I\left(\int_a^b d\vec{s}\right) \times \vec{B}_0 = I\vec{L} \times \vec{B}_0$$



$$\vec{F}_2 = \vec{F}_4 = 0$$

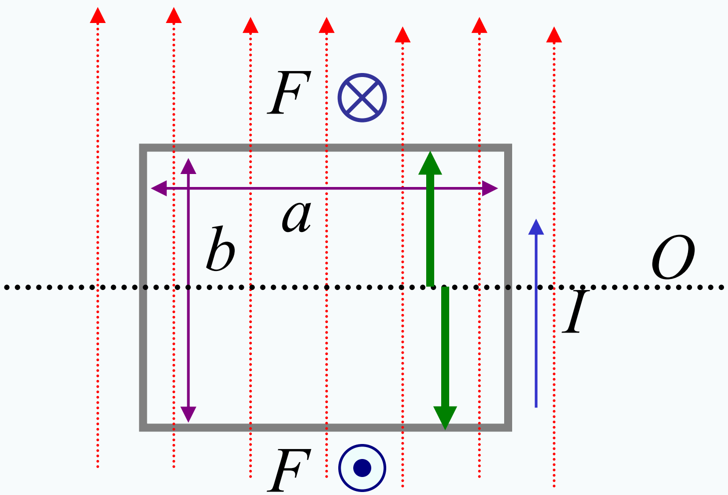
$$\vec{F}_1 = ILB_0 = -\vec{F}_3$$

$$\sum_{loop} \vec{F}_i = 0$$



Maximal torque on a loop

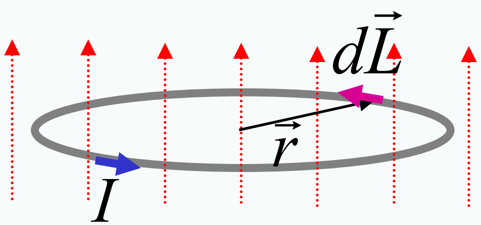
$$F = IaB_0$$



$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d(\vec{r} \times \vec{p})}{dt} = \vec{r} \times \vec{F}$$

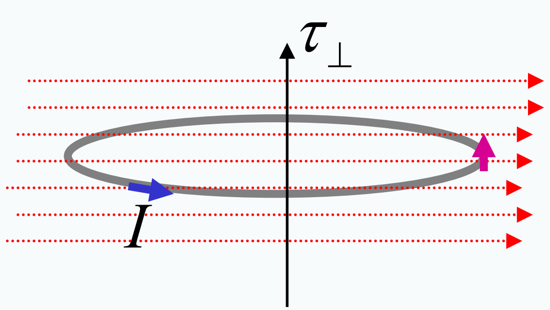
$$\tau_{net} = \tfrac{1}{2} bF + \tfrac{1}{2} bF = IabB_0 = IAB_0$$

$$\vec{F}_B = I(\vec{L} \times \vec{B}) \qquad \vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$



$$\vec{\tau} = I\left[\vec{L}(\vec{r} \cdot \vec{B}) - \vec{B}(\vec{r} \cdot \vec{L})\right]$$

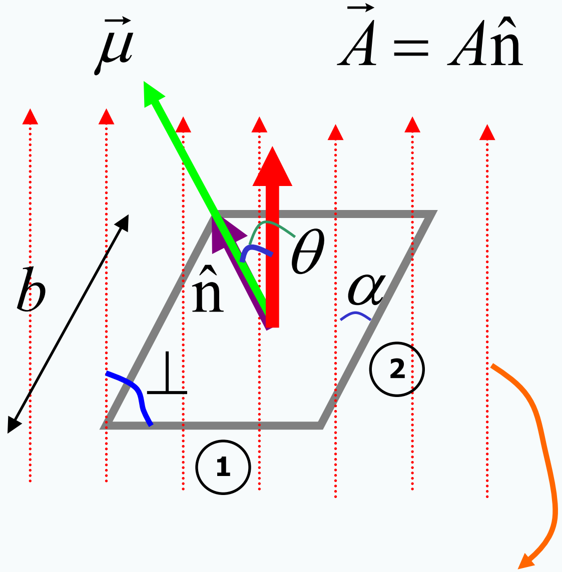
$$d\vec{\tau} = I\left[d\vec{L}(\vec{r} \cdot \vec{B}) - \vec{B}(\vec{r} \cdot d\vec{L})\right]$$



$$d\vec{\tau} = I\left[d\vec{L}(\vec{r} \cdot \vec{B}) - \vec{B}(\vec{r} \cdot d\vec{L})\right]$$

$$d\tau_\perp = IBR\int \cos\alpha \, dL_\perp = IBR^2 \int_0^{2\pi} \cos^2\alpha \, d\alpha = IB\pi R^2$$

Torque – generalized equation



$$\vec{A} = A\hat{n}$$

$$\tau_{net} = bF\cos\alpha$$

$$\tau_{net} = bF\sin\theta$$

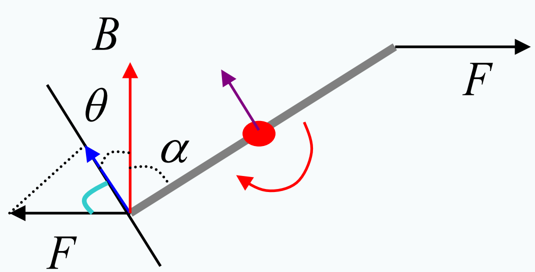
$$F = IaB$$

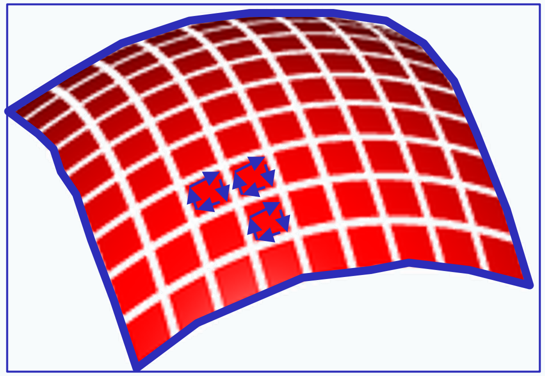
$$\vec{\tau} = I\vec{A}\times\vec{B}$$

The direction of the normal to the surface is determined using right-hand rule by curling the fingers in the direction of the electric current.

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$





$$\vec{\tau} = \int d\vec{\mu}\times\vec{B}$$

$$d\vec{\mu} = Id\vec{A}$$

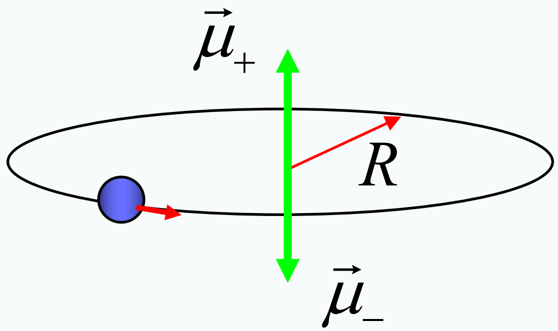
$$\vec{A} = \int d\vec{A}$$

$$dW_B = \tau d\theta = \tau_0 \sin\theta d\theta$$

$$dW_B = -dU$$

$$U = -\mu B\cos\theta = -\vec{\mu}\cdot\vec{B}$$

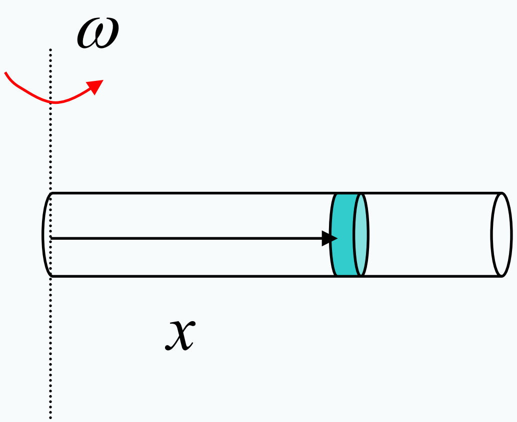
Magnetic moments



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\vec{\mu} = I\vec{A} \qquad \vec{\mu} = \frac{q}{T}\pi R^2 \hat{n} = \frac{1}{2} q\omega R^2 \hat{n}$$

$$\vec{L} = \omega \vec{I} = m\omega R^2 \hat{n} \qquad \vec{\mu} = \frac{1}{2}\frac{q}{m}\vec{L}$$

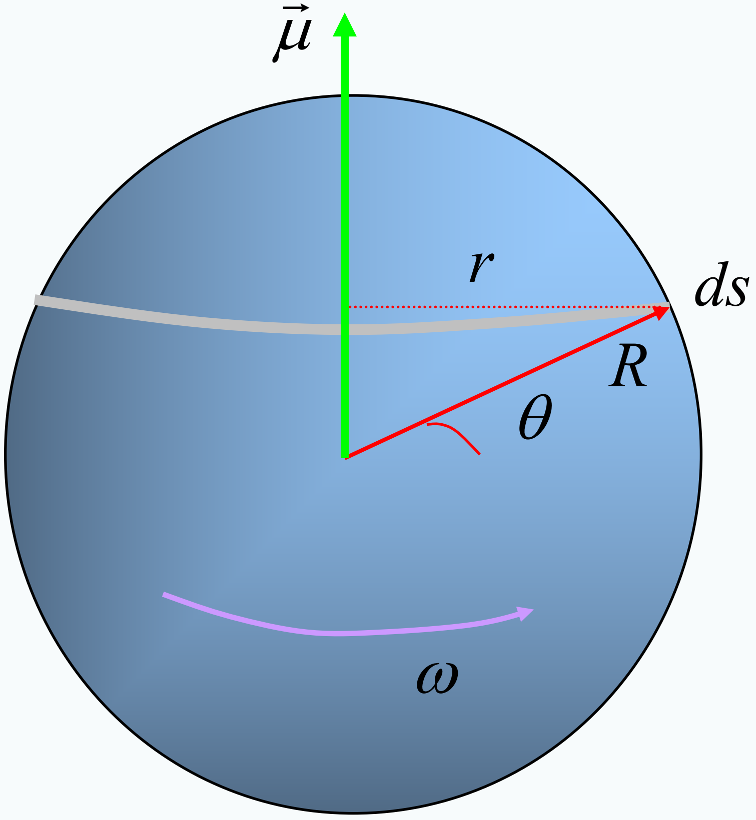


$$dI = \frac{\pi r^2 dx \rho \omega}{2\pi} \qquad A = \pi x^2$$

$$\mu = \frac{\pi r^2 \rho \omega}{2} \int_0^L x^2 dx = \frac{Q\omega L^2}{6}$$

$$I = \int_0^L x^2 dm = \rho \pi r^2 \int_0^L x^2 dx = \frac{1}{3} M L^2 \qquad \vec{\mu} = \frac{1}{2}\frac{Q}{M}\vec{L}$$

Magnetic moment of a spherical shell



$$d\mu = AdI \qquad dI = \frac{dQ}{T}$$

$$dQ = \sigma \cdot 2\pi r ds$$

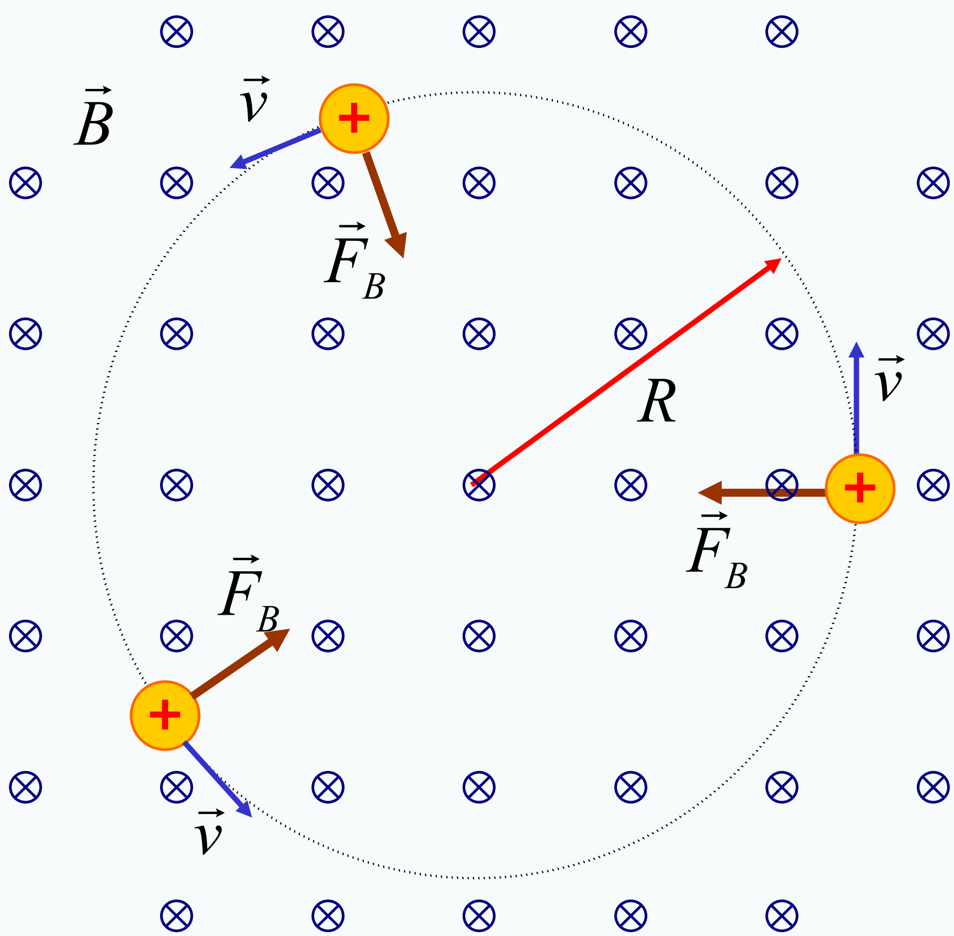
$$dQ = \sigma \cdot 2\pi R\cos\theta \cdot Rd\theta$$

$$dI = \sigma\omega R\cos\theta \cdot Rd\theta$$

$$d\mu = \pi R^4 \cos^3\theta \cdot \sigma\omega d\theta$$

$$\mu = 2\pi\sigma\omega R^4 \int_0^{\pi/2} \cos^3\theta d\theta = \frac{4}{3}\pi\sigma\omega R^4 = \frac{1}{3}Q\omega R^2$$

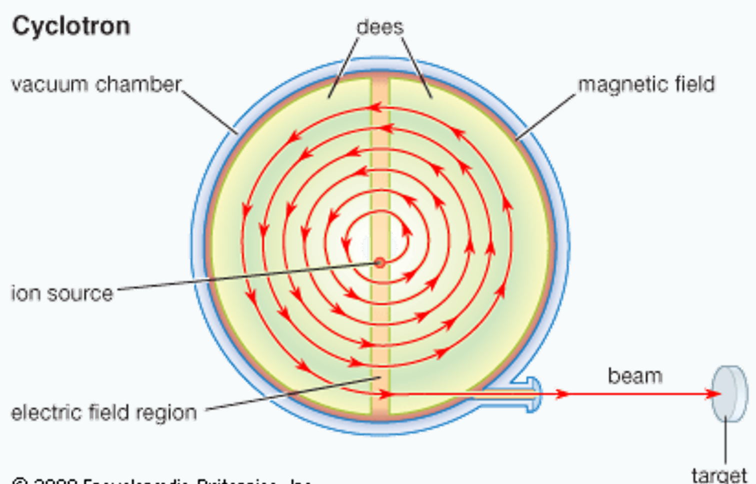
Charge moving perpendicular to B



$$\sum \vec{F} = m\vec{a}$$

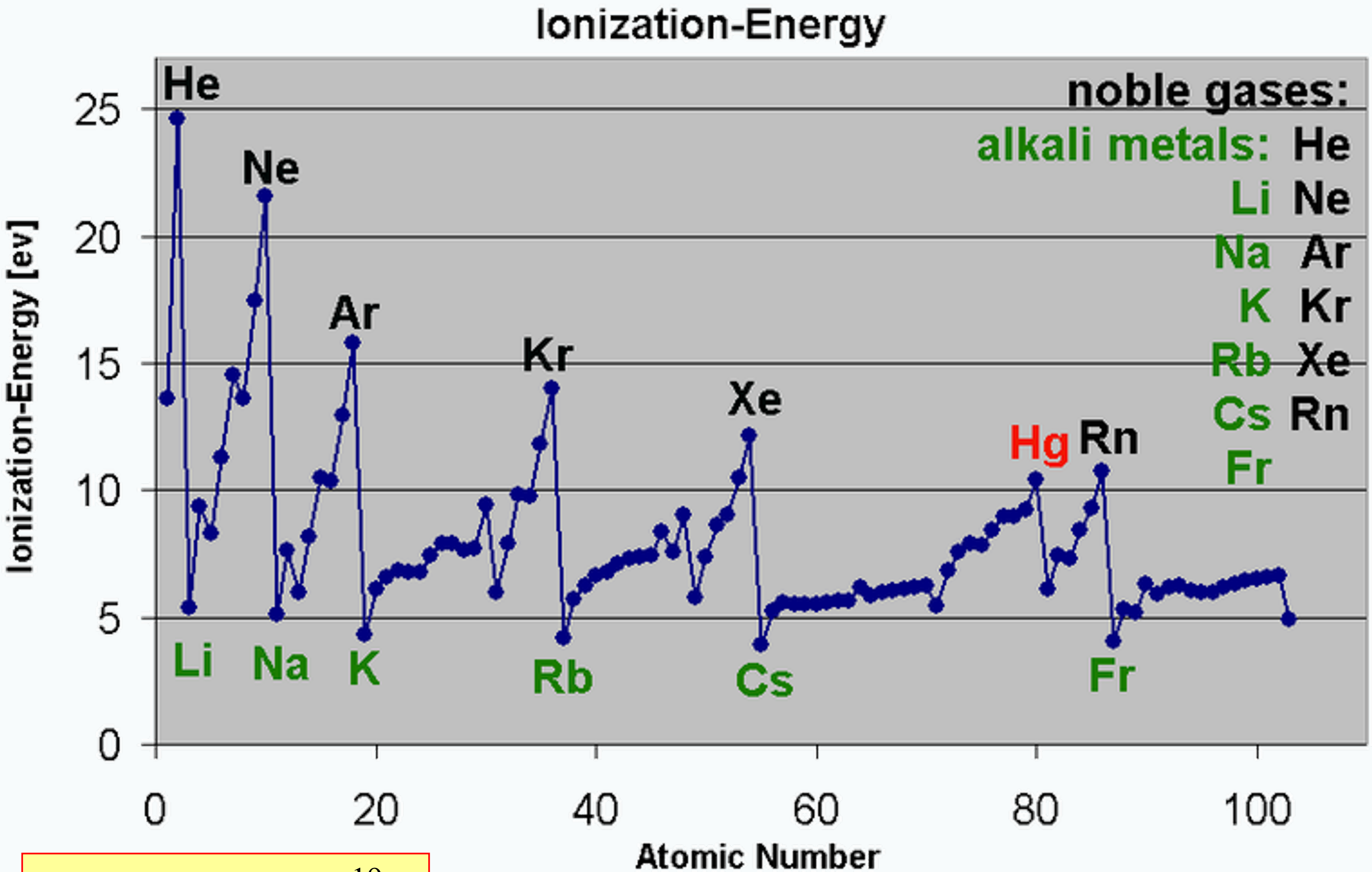
$$F_B = qvB = ma_\perp$$

$$R = \frac{mv}{qB}$$



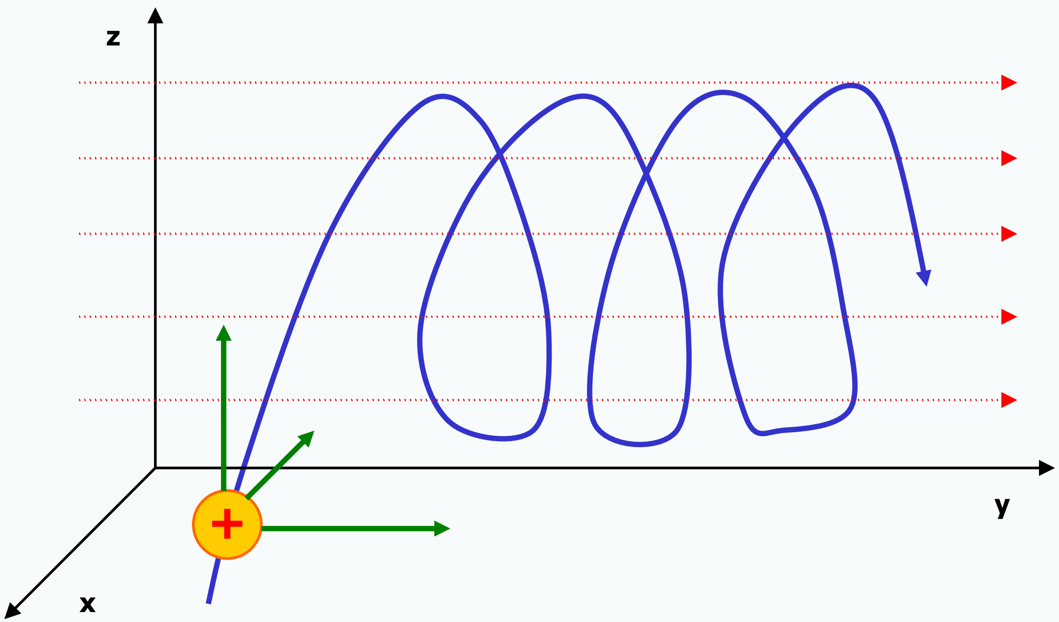
$$\omega = \frac{qB}{m}$$ **Cyclotron frequency**

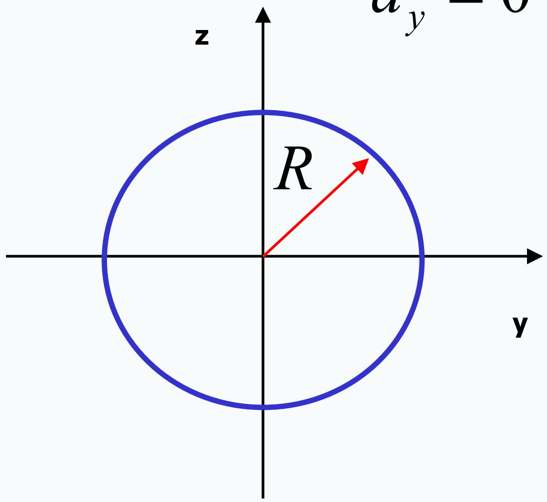
Ionization energies



$$1\ \mathrm{eV} \approx 1.6 \times 10^{-19}\ \mathrm{J}$$

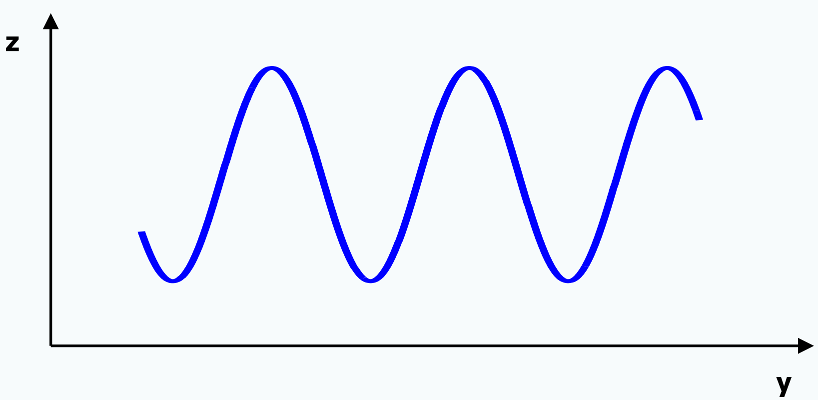
Arbitrary angle





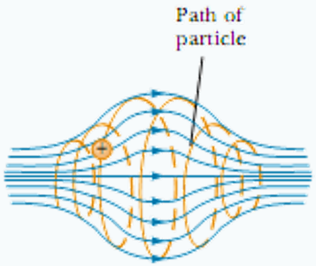
$$a_y = 0$$

$$R = \frac{m|v_{zx}|}{qB} = \frac{m}{qB}\sqrt{v_x^2 + v_z^2}$$

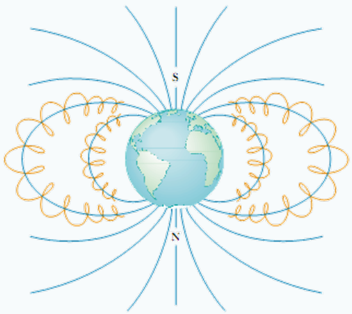


$$\begin{cases} v_y = const \\ v_x = v_0 \cos\alpha \\ v_z = v_0 \sin\alpha \end{cases} \qquad \begin{cases} y = v_y t \\ x = v_0 t \cos y \\ z = v_0 t \sin y \end{cases}$$

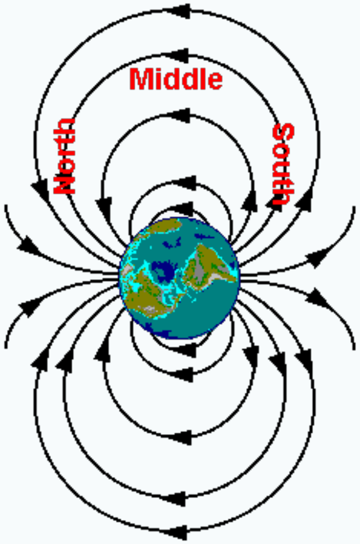
Some examples

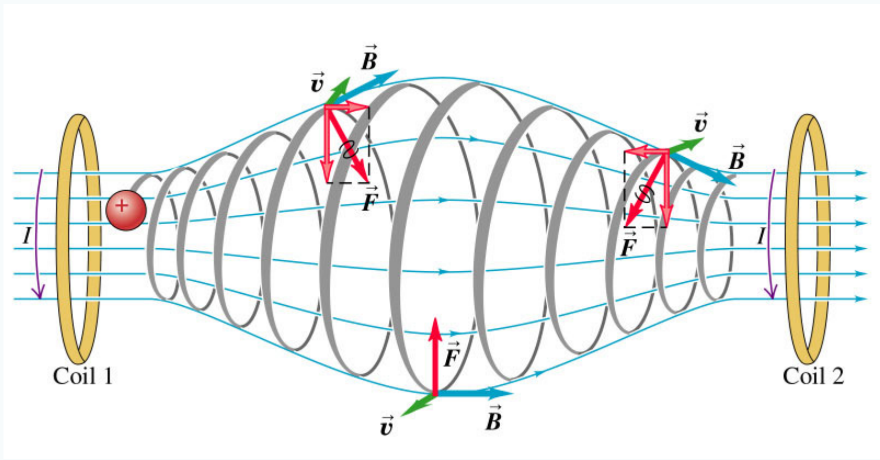


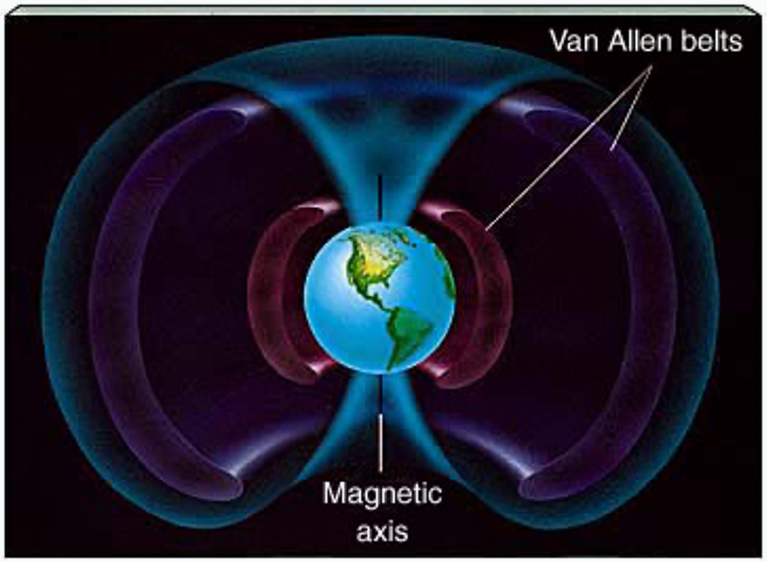
Magnetic bottle



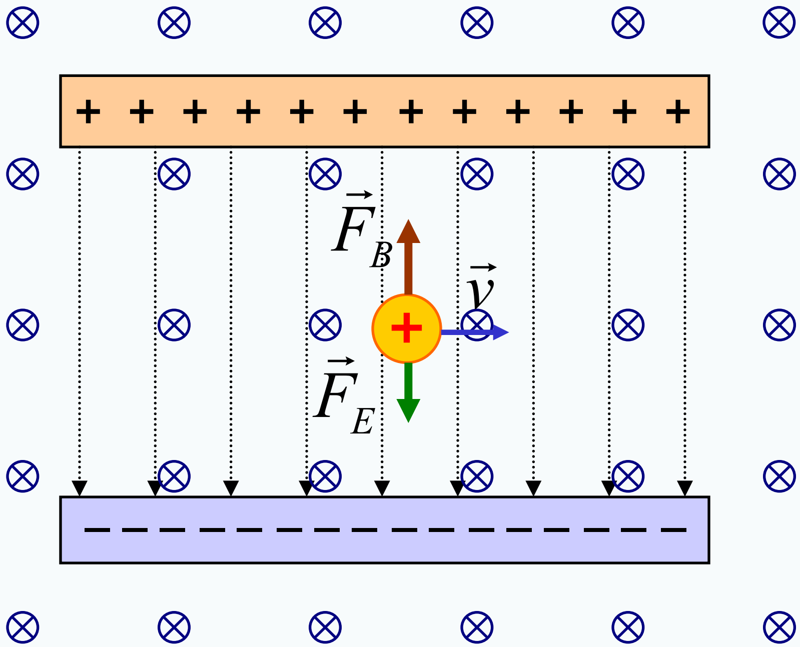
Van Allen belts







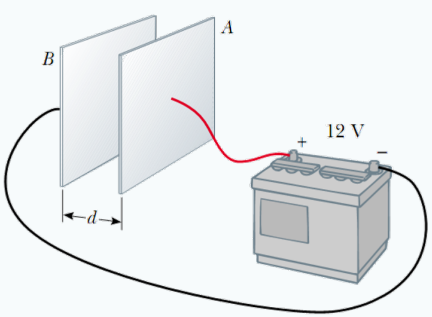
Velocity selector



$$\vec{F}_{net} = q\vec{v} \times \vec{B} + q\vec{E}$$

$$0 = qvB - qE$$

$$v_{\text{unaffected}} = \frac{E}{B}$$



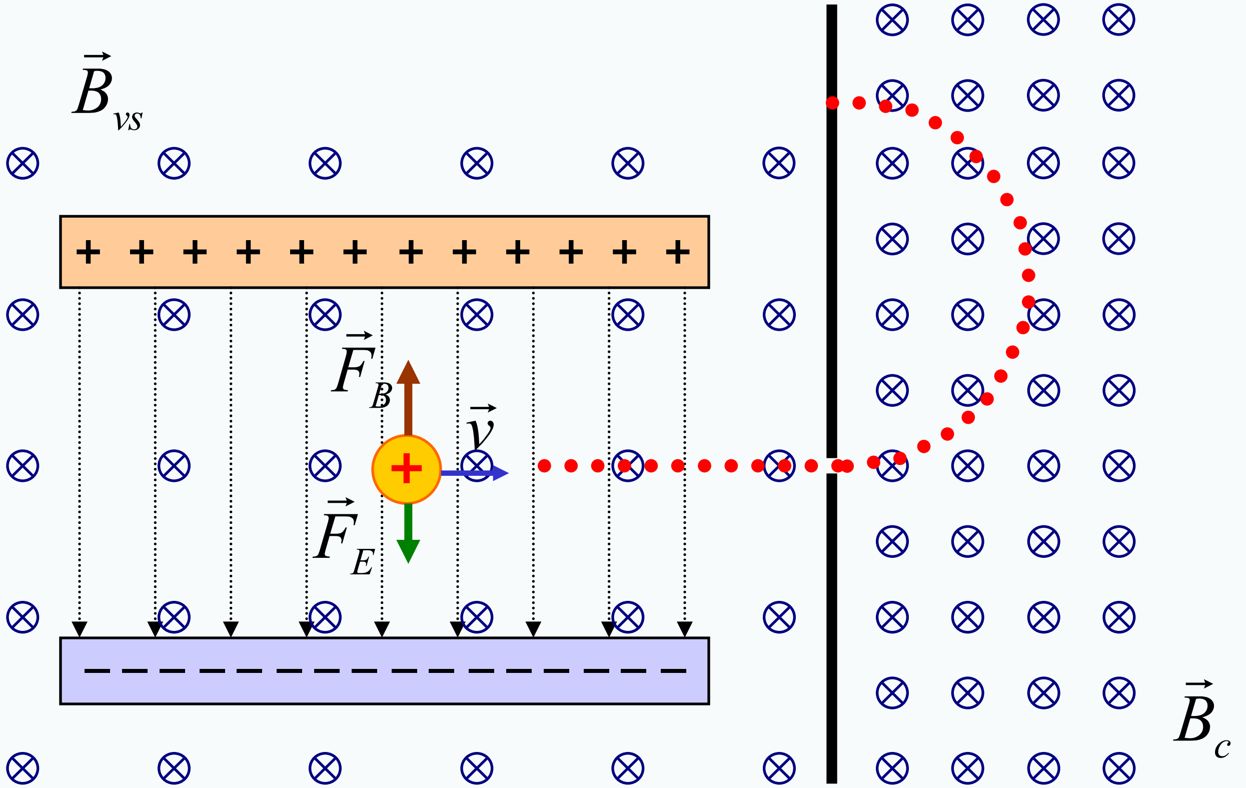
$$d = 1\,\text{mm} - 1\,\text{m}$$

$$B = 12\ \text{T}$$

$$v_{\text{unaffected}} = \frac{V}{dB}$$

$$v_{\text{unaffected}} = 1\,\text{km/s} - 1\,\text{m/s}$$

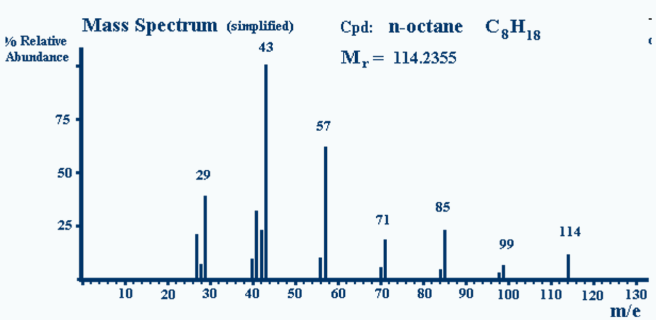
Mass spectrometer

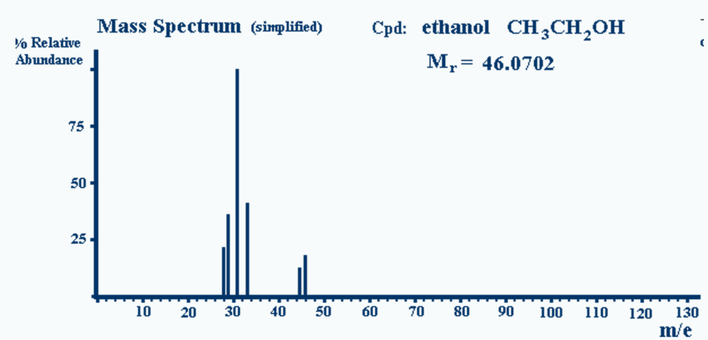


$$R = \frac{mv}{qB_c}$$

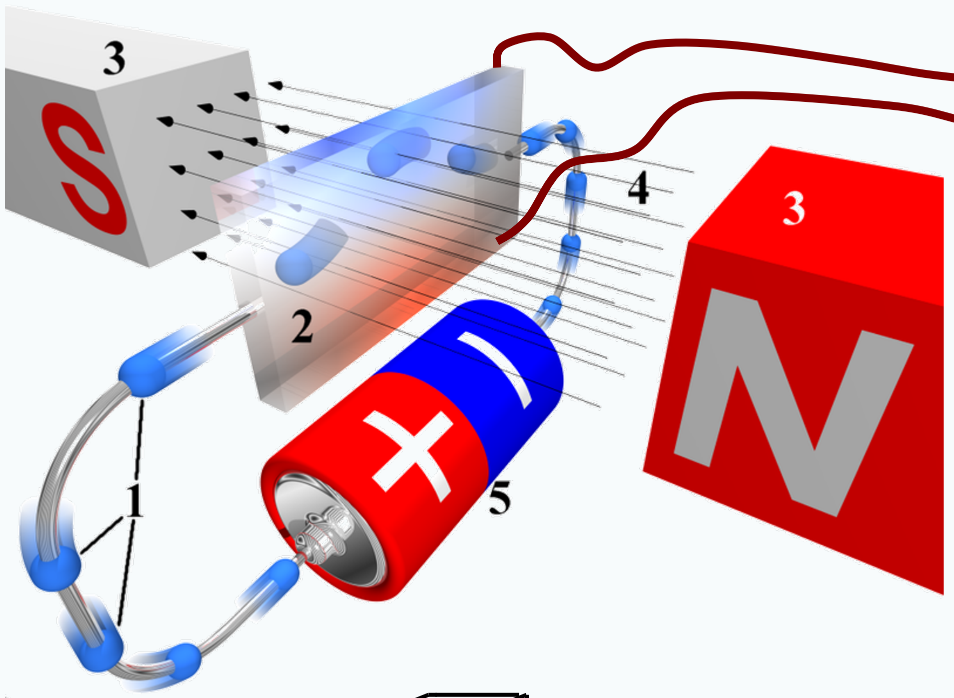
$$m = \frac{qB_c L}{2v}$$

$$m = \frac{qB_{vs} B_c L}{2E}$$





Hall effect



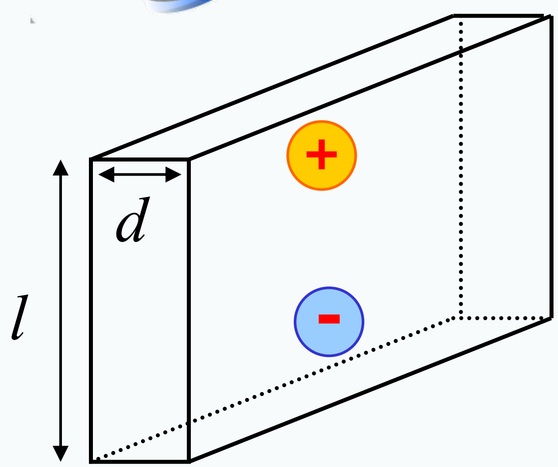
Hall voltage

In equilibrium:

$$qv_dB = qE_{\text{Hall}}$$

$$E_{\text{Hall}} = v_dB$$

$$\Delta V_{\text{Hall}} = l \cdot E_{\text{Hall}} = lv_dB$$

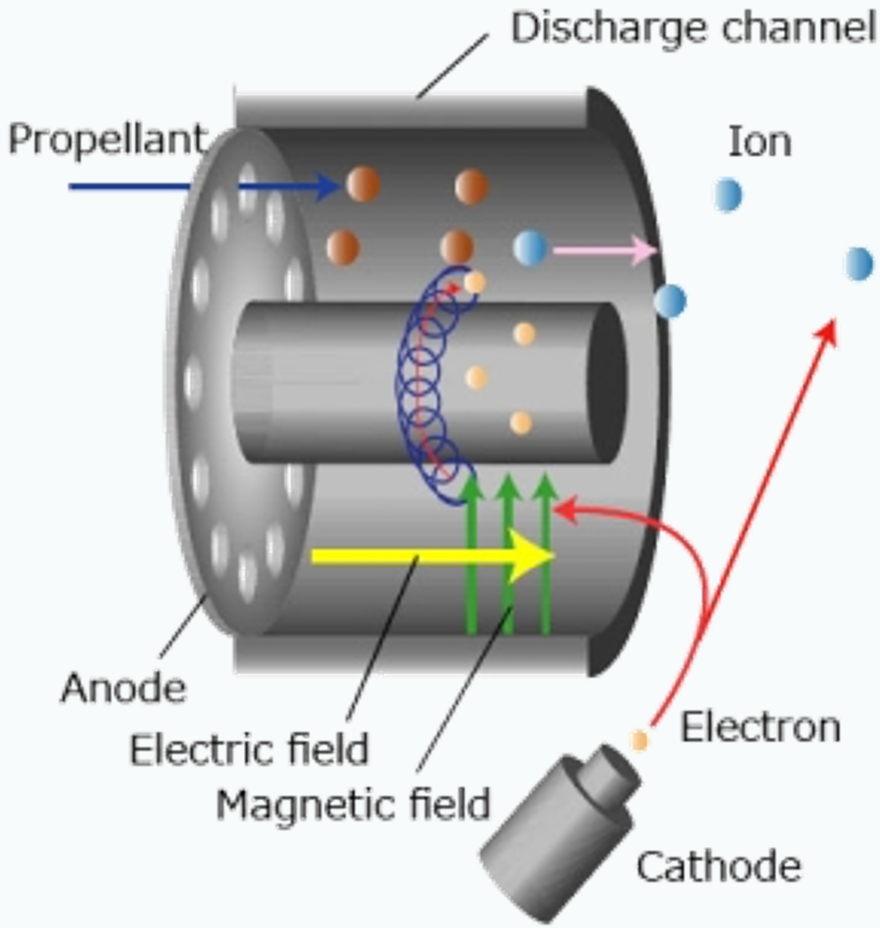


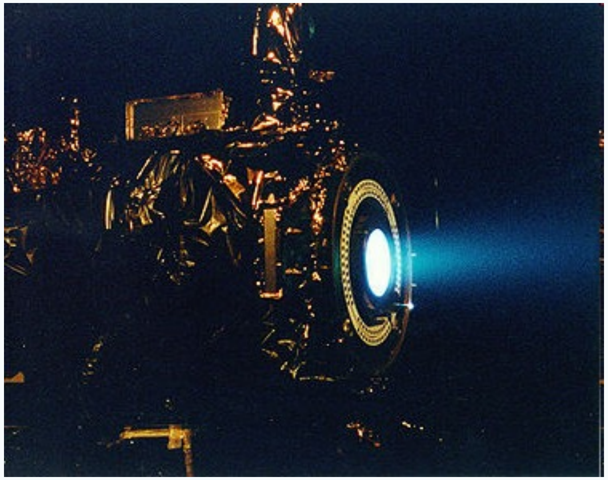
$$I = nqv_d dl \qquad \Delta V_{\text{Hall}} = \frac{I}{nqd}B \equiv R_H \frac{IB}{d}$$

Silver: $R_H = 0.84 \times 10^{-10}\ \text{m}^3/\text{C}$

Hall coefficient

Hall effect thruster





To remember!

- Magnetic force acts only upon moving charges.
- Magnitude of magnetic field is measured in Tesla.
- The direction of this force perpendicular to both the velocity of the particles and to the magnetic field.
- The magnitude is proportional to the charge, the speed, the magnetic field and the sine of the angle between v and B.
- The same applies to unidirectional conductors, but with charge replaced by the electric current.
- For curved conductors, the total force is found by integrating along the conductor.
- No net magnetic force acts upon a closed-loop conductor placed in an uniform magnetic field.

To remember!

- **Magnetic force acting upon a curved conducting wire carrying electric current is found by dividing it on small straight parts and summing up (integrating over) the forces acting upon them.**
- **Homogeneous magnetic filed exert no force on any current loop.**
- **However, it can exert torque.**
- **The toque is maximal when the magnetic filed is parallel to the plane of a flat current loop and zero it is perpendicular to it.**
- **Magnetic moment of a current loop is product of the electric current and the loop area.**
- **Any charged rotating object, i.e. having non-zero angular momentum, has as well magnetic moment.**

To remember!

- **If a charged particle enters magnetic field with initial velocity perpendicular to the field lines, it will get trapped by the magnetic field and perform circular motion.**
- **The angular frequency is called cyclotron frequency.**
- **For other angles the trajectory is a helix.**
- **Simultaneous application of electric and magnetic fields allows velocity selection of charged molecules.**
- **With one more magnetic field mass spectrometer can be constructed.**
- **In conductors carrying electric current and placed in magnetic field the Hall potential difference is developed due to deflection of moving electrons.**