Experimental Physics EP2a

Electricity and Wave Optics

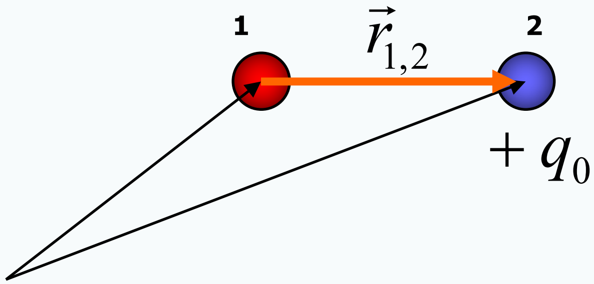
– Electric potential –
– Capacitors –

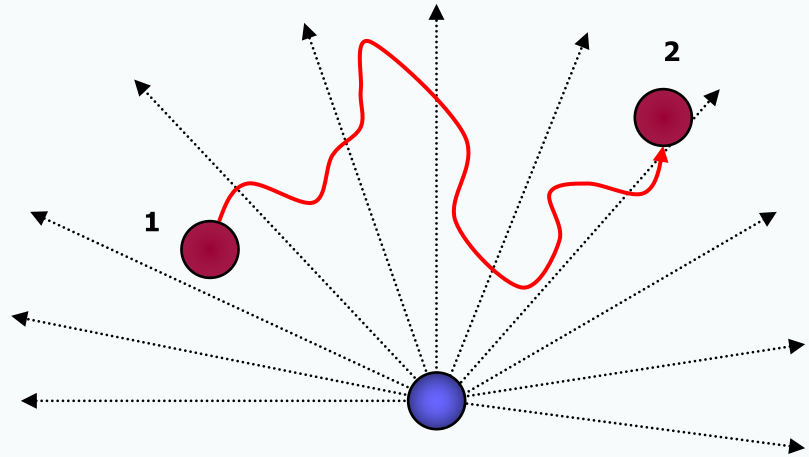
https://bloch.physgeo.uni-leipzig.de/amr/

Electric potential

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$





$$W_{\text{by field}} = \int \vec{F}d\vec{s} = \int q_0\vec{E}d\vec{s} \quad = -\Delta U$$

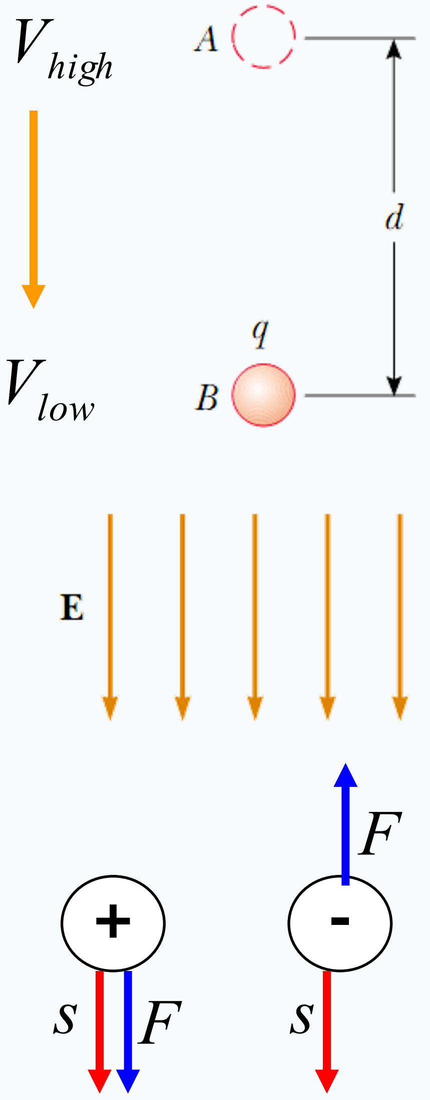
change of potential energy

$$\Delta V = \frac{\Delta U}{q_0}$$ **- potential difference**

$$\Delta V = -\int_1^2 \vec{E}d\vec{s} \qquad \left[\frac{J}{C}\right]$$

$$\vec{E} \qquad \left[\frac{V}{m}\right]$$

Constant electric field



$$\Delta V = -\int_A^B \vec{E} d\vec{s} \quad = -E\int_A^B ds \quad = -Ed$$

$$\Delta V = V_B - V_A < 0$$

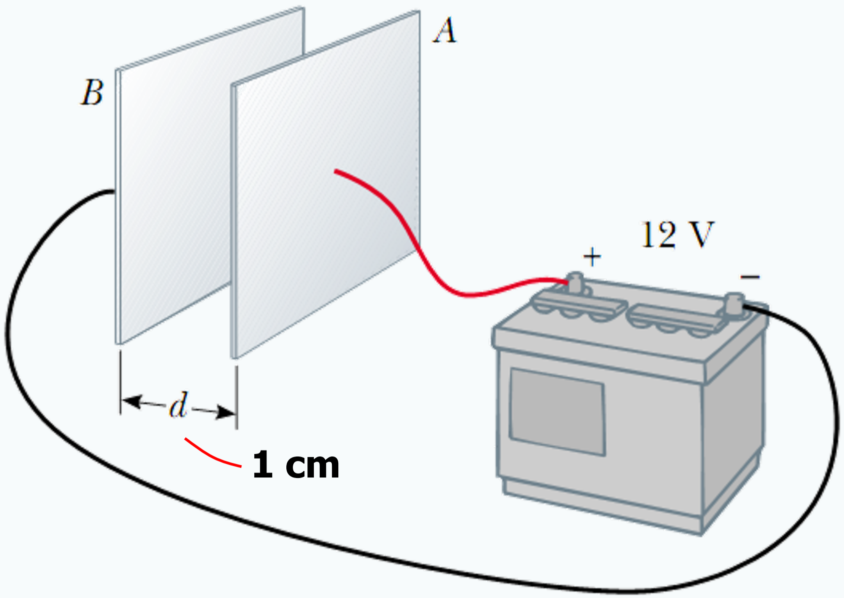
$$\boxed{\Delta U = q_0 \Delta V} \qquad q_0 > 0 \quad \Rightarrow \quad \Delta U < 0$$

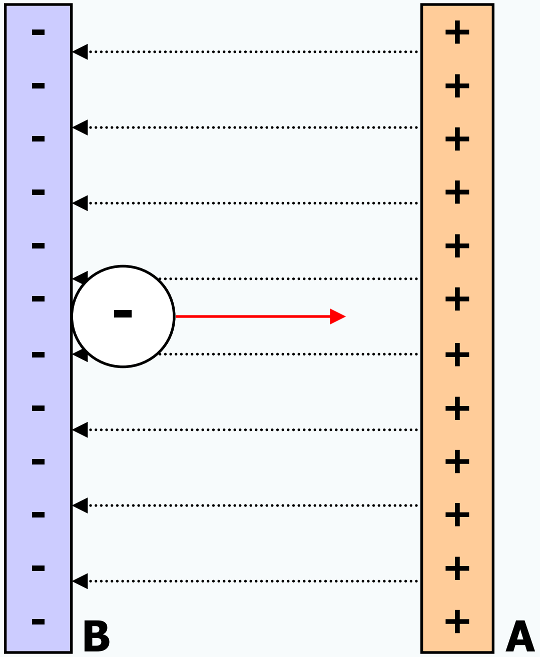
- **A positive charge loses electric potential energy when it moves in the direction of the electric field.**

- **A negative charge gains electric potential energy when it moves in the direction of the electric field.**

Equipotential surface **is any surface along which the electric potential is identical.**

Selected examples





$$E - ?$$

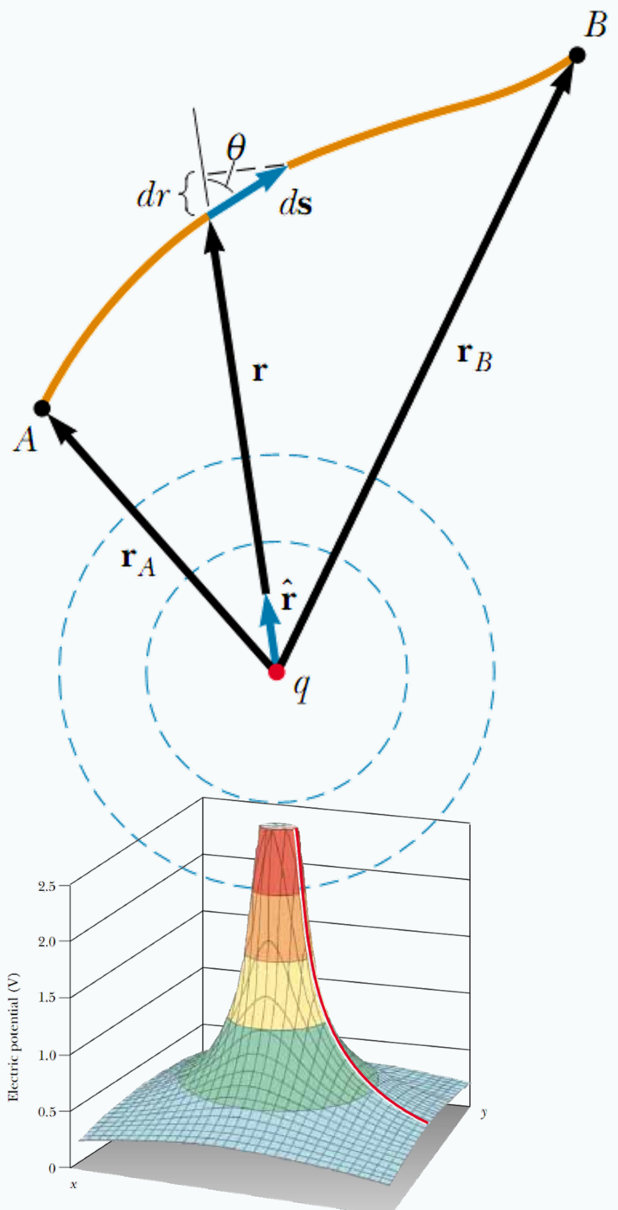
$$E = \frac{|V_A - V_B|}{d} = 1.2\ \mathrm{kV/m}$$

$$v_A - ?\ \text{if}\ v_B = 0$$

$$\Delta U = q_0 \Delta V \quad = 19.2 \times 10^{-19}\ \mathrm{J}$$

$$v_A = \sqrt{\frac{2\Delta U}{m}} \quad \approx 1.7 \times 10^4\ \mathrm{m/s}$$

Potential of a point-like charge



$$\Delta V = -\int_A^B \vec{E} d\vec{s}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r} \qquad \vec{E}\cdot d\vec{s} = \frac{kq}{r^2}\hat{r}\cdot d\vec{s}$$

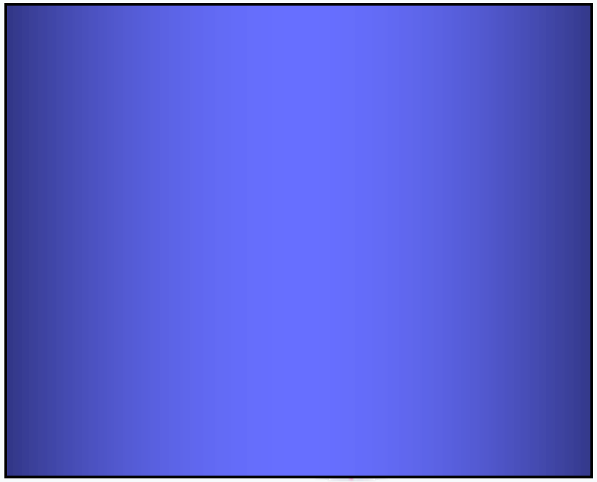
$$\hat{r}\cdot d\vec{s} = ds\cos(\theta) = dr$$

$$\Delta V = -\int_{r_A}^{r_B} \frac{kq}{r^2} dr = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

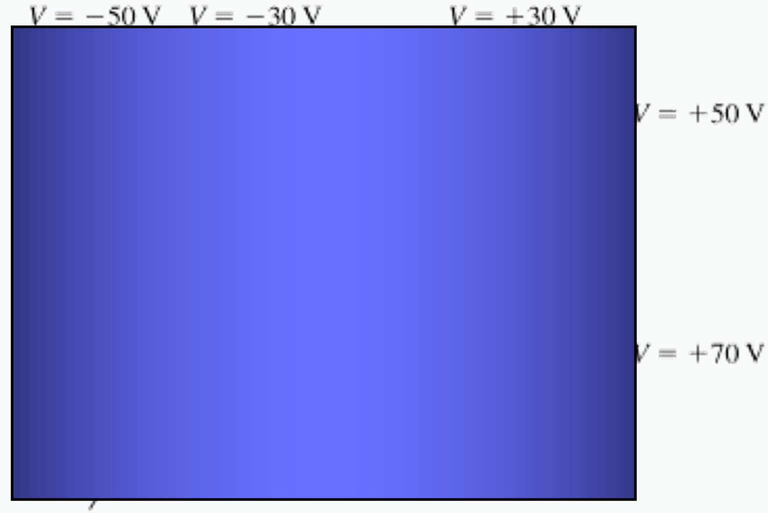
$r_A \to \infty$

$$V = \frac{kq}{r}$$

Equipotential surfaces



(a) A single positive charge

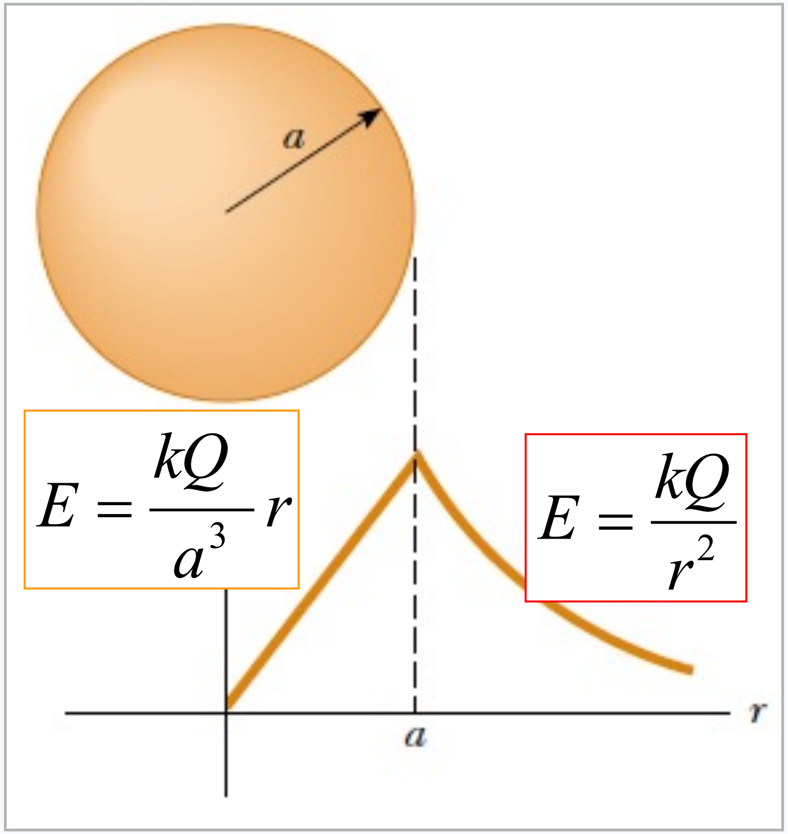


(b) An electric dipole

(c) Two equal positive charges

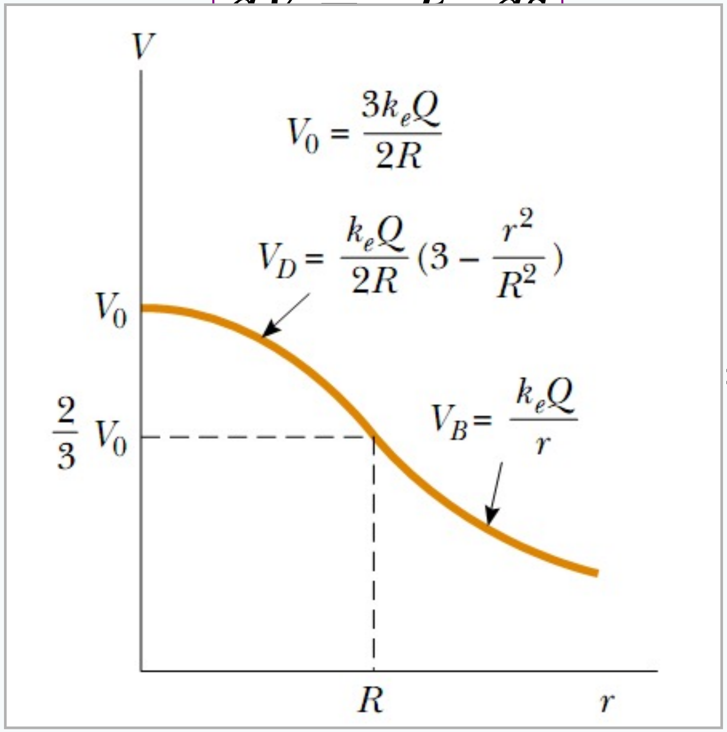
$$dV = -\vec{E} \cdot d\vec{s}$$

Insulators: Plastic ball



$$E = \frac{kQ}{a^3} r$$

$$E = \frac{kQ}{r^2}$$

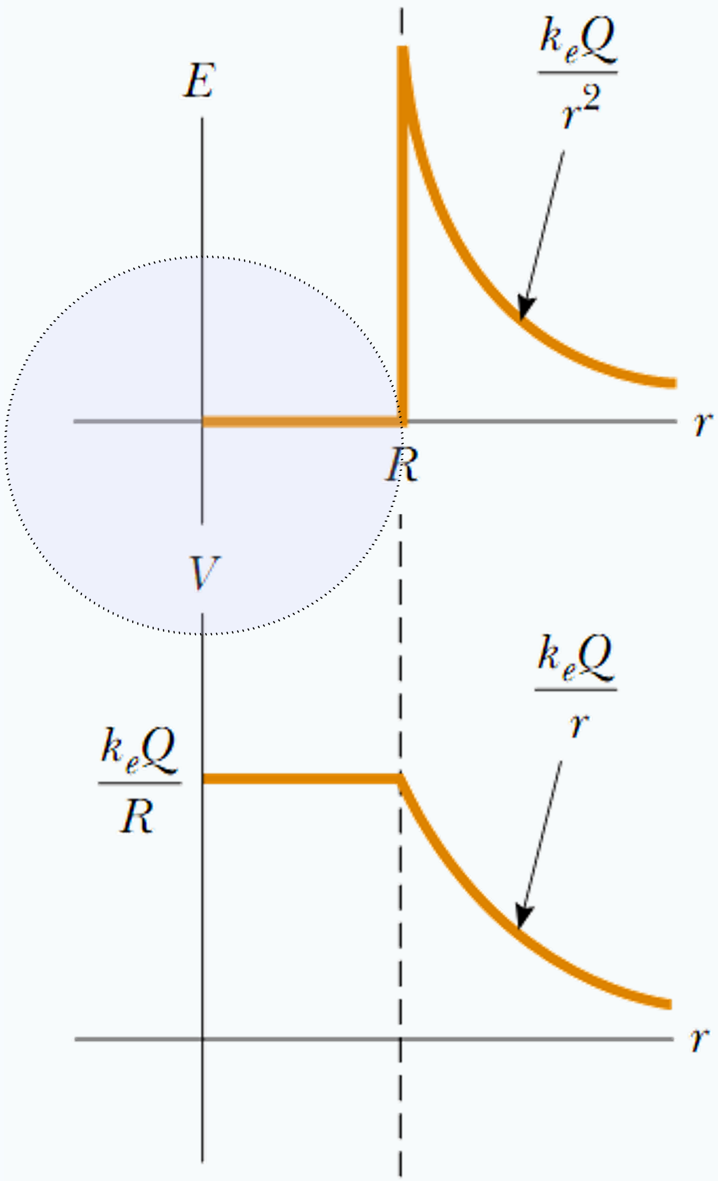


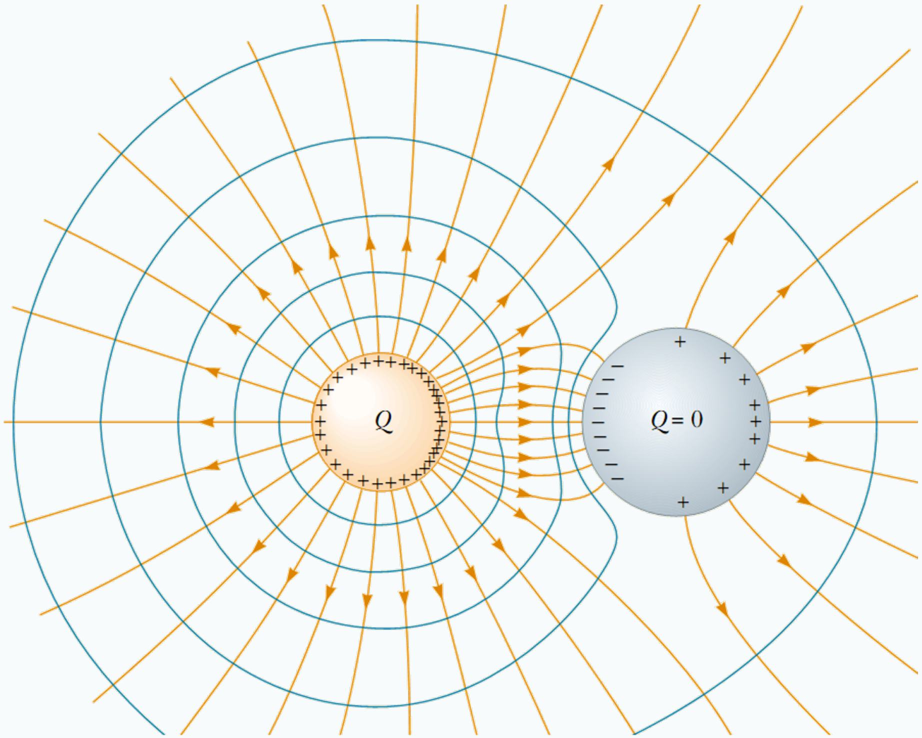
$$= \frac{kQ}{r}$$

$$V_{in} - V_{surface} = -\int_a^r \frac{kQ}{a^3} r\,dr = -\frac{kQ}{2a^3}\left(r^2 - a^2\right)$$

$$V_{in} = \frac{kQ}{2a}\left(3 - \frac{r^2}{a^2}\right)$$

Conductors





Surface of any conductor at electrostatic equilibrium is equipotential surface.

Two conductors in contact

Surface of any conductor at electrostatic equilibrium is equipotential surface.

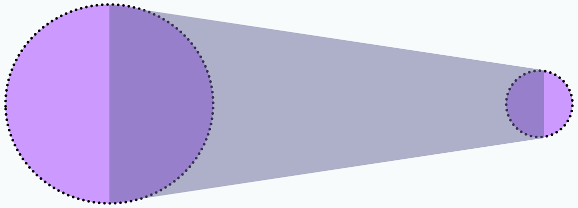
$$Q_R + Q_r = Q$$

$$\frac{kQ_R}{R} = \frac{kQ_r}{r}$$

$$\frac{Q_R}{Q_r} = \frac{R}{r}$$

$$V = \frac{kQ_R}{R} = 4\pi kR\sigma$$

$$E = 4\pi k\sigma$$

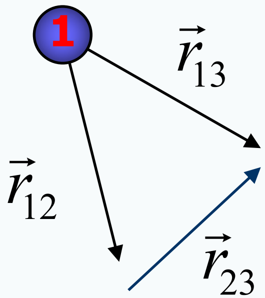


$$\sigma_R = \frac{V}{4\pi kR} \qquad \sigma_r = \frac{V}{4\pi kr}$$

To remember!

- When a charge is moved from point A to point B, the decrease of its potential energy is equal to work done by the electric field.
- Electric potential is a scalar quantity equal to the electric potential energy per unit charge.
- Only changes in electric potential are important.
- It is convenient to take the reference point at infinity.
- An equipotential surface is one on which all points are at the same electric potential.
- Equipotential surfaces are perpendicular to electric field lines.
- Surfaces of conductors are equipotential surfaces.

Electrostatic energy



$$V_2 = \frac{kq_1}{r_{12}} \qquad W_2 = q_2 \frac{kq_1}{r_{12}}$$

$$V_3 = \frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}} \qquad W_3 = q_3\left(\frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}}\right)$$

$$W = U = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

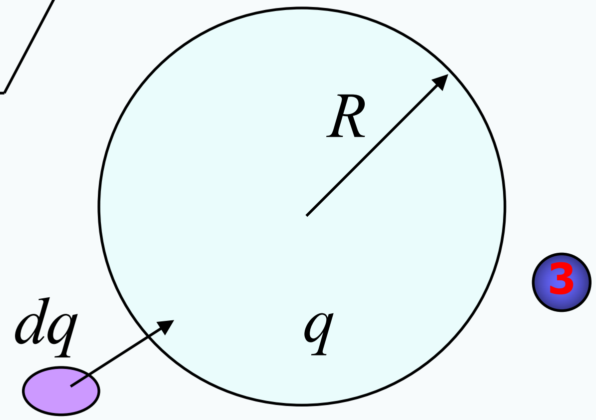
Electrostatic potential energy of a system of charges is the work needed to bring all particles together from an infinite separation.

$$U = \frac{1}{2}q_1V_1 + \frac{1}{2}q_2V_2 + \frac{1}{2}q_3V_3 = \sum_i \tfrac{1}{2}q_iV_i$$

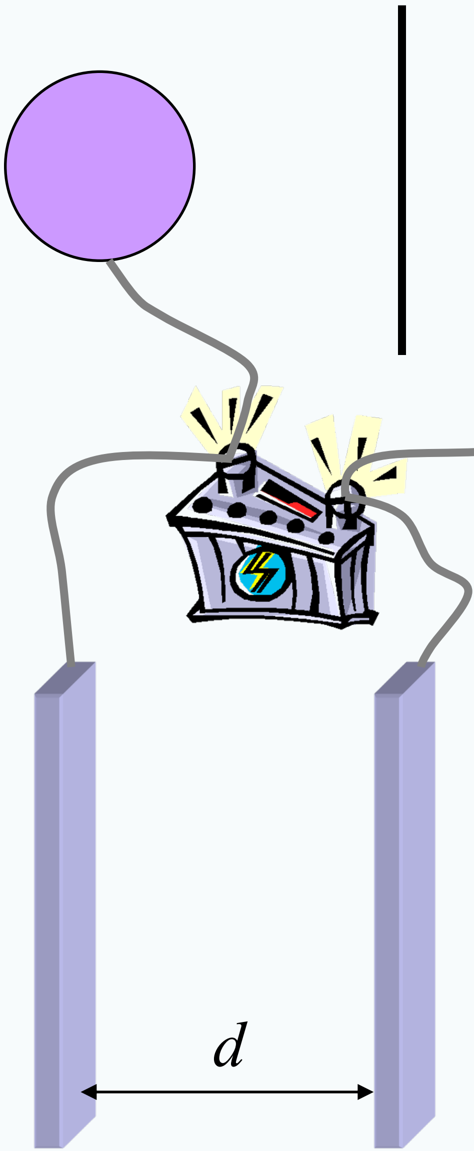
$$dU = Vdq = \frac{kq}{R}dq$$

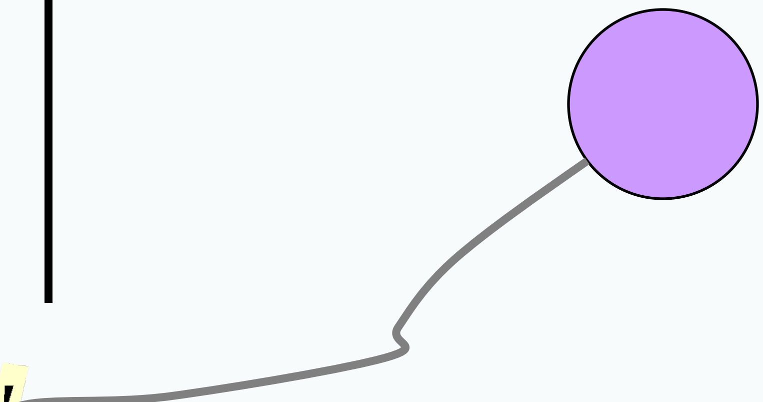
$$U = kR^{-1}\int_0^Q qdq = \tfrac{1}{2}kR^{-1}Q^2$$

$$U = \tfrac{1}{2}QV$$



Capacitance





$$V = \frac{kQ}{R}$$

$$C = \frac{R}{k}$$

$$C \equiv \frac{Q}{\Delta V} \qquad \left[\frac{C}{V} \equiv F\right]$$

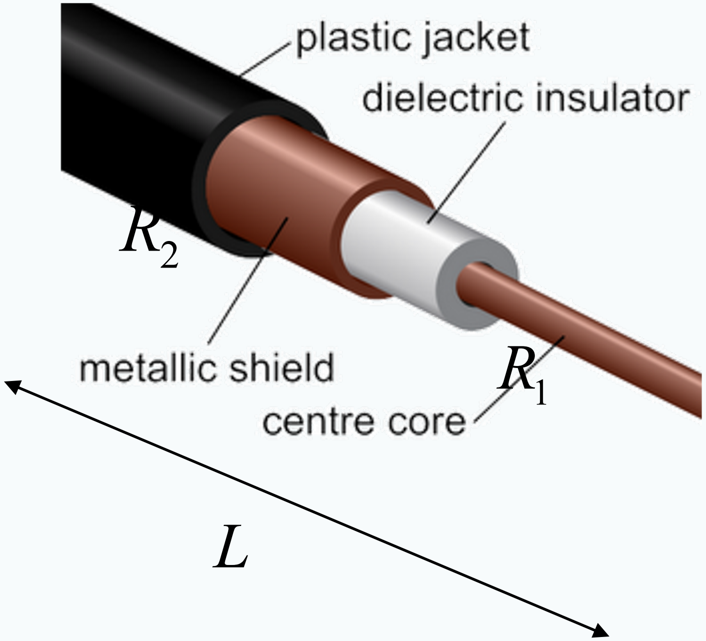
$$E = 4\pi k\sigma \qquad V = Ed = 4\pi k\frac{Q}{A}d$$

$$C = \frac{A}{4\pi kd}$$

Capacitance does not depend on the charge or the electric potential.

Capacitance does depend on the configuration (size, shape, separation, geometrical arrangement) of the conductors.

Capacitance of a coaxial cable



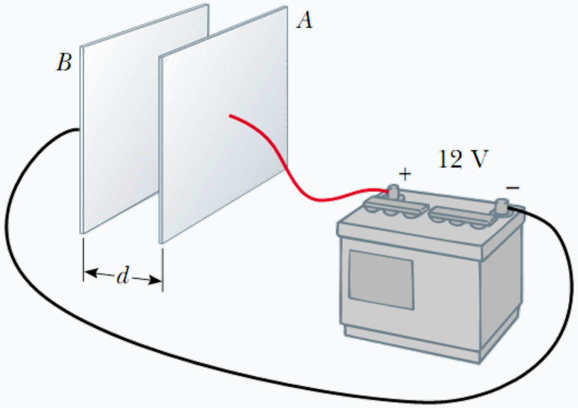
$$E \cdot 2\pi r L = 4\pi k Q$$

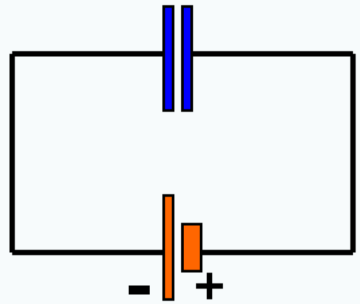
$$\Delta V = -\int_{R_1}^{R_2} \vec{E} d\vec{r} = -\int_{R_1}^{R_2} \frac{2kQ}{Lr} dr$$

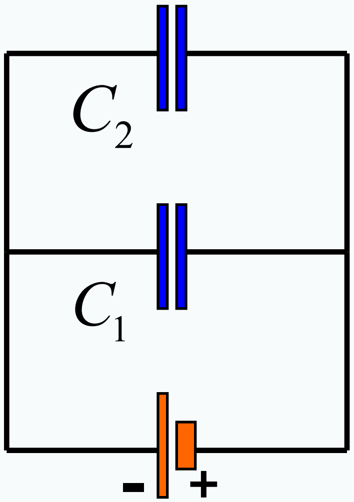
$$|\Delta V| = \frac{2kQ}{L} \ln\left(\frac{R_2}{R_1}\right)$$

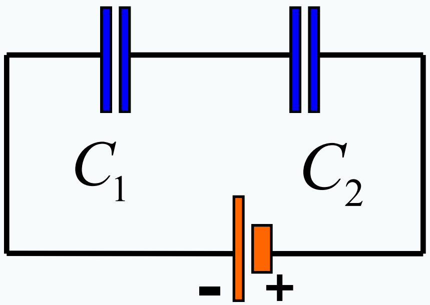
$$C = \frac{Q}{|\Delta V|} = \frac{L}{2k \ln\left(R_2/R_1\right)}$$

Circuits



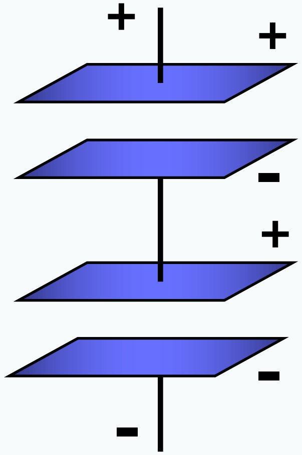






$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{V}{Q} = \frac{V_1 + V_2}{Q}$$



$$C_{ser}^{-1} = \sum C_i^{-1}$$

$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

$$C = \frac{Q}{V} = \frac{Q_1 + Q_2}{V}$$

$$C_{\parallel} = \sum C_i$$

Energy stored in a capacitor

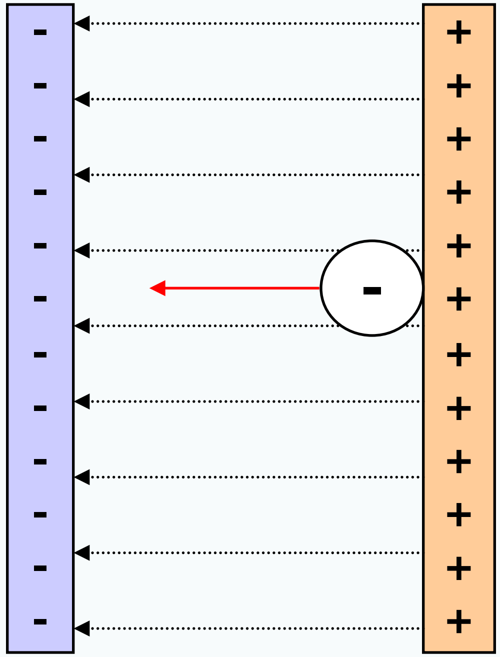
$$dW = \Delta V dq \qquad W = \int_0^Q \Delta V dq$$

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C} \quad = \frac{1}{2} C (\Delta V)^2$$

$$\Delta V = Ed \qquad C = \frac{A}{4\pi k d} = \frac{A\varepsilon_0}{d}$$

$$W = \frac{1}{2} Ad \varepsilon_0 E^2 = u_E A d$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$ **- energy density, is applicable for any field**



Dielectrics

$$E = \frac{E_0}{\kappa}$$

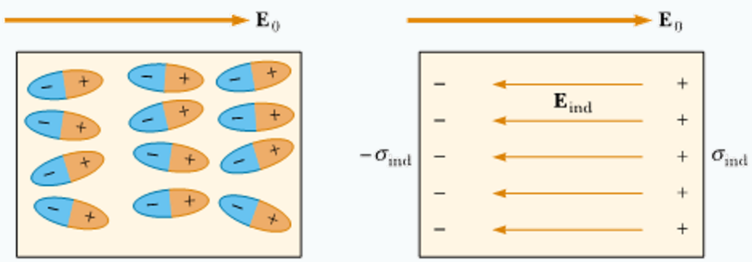
dielectric constant

permittivity

$$\varepsilon = \varepsilon_0 \kappa$$

$$V = \frac{V_0}{\kappa}$$

$$C = C_0 \kappa$$



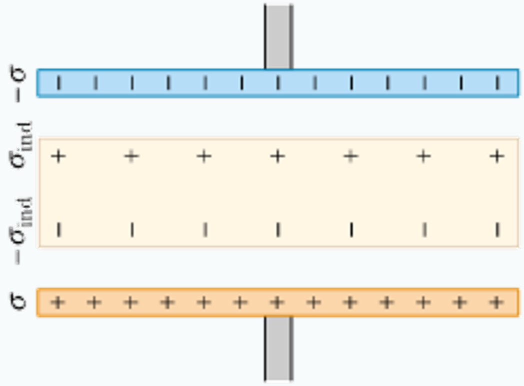


TABLE 26.1 Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (V/m)
Air (dry)	1.000 59	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Polyvinyl chloride	3.4	40×10^6
Porcelain	6	12×10^6
Pyrex glass	5.6	14×10^6
Silicone oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Vacuum	1.000 00	—
Water	80	—

Electric displacement field

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = \varepsilon_0(\kappa - 1)\vec{E}$$

- $\vec{P}$: polarization density (per unit volume)
- $\varepsilon_0 \chi_e$: permittivity
- κ: relative permittivity; dielectric constant

$$\oint \vec{D} d\vec{A} = q_{enclosed,free}$$

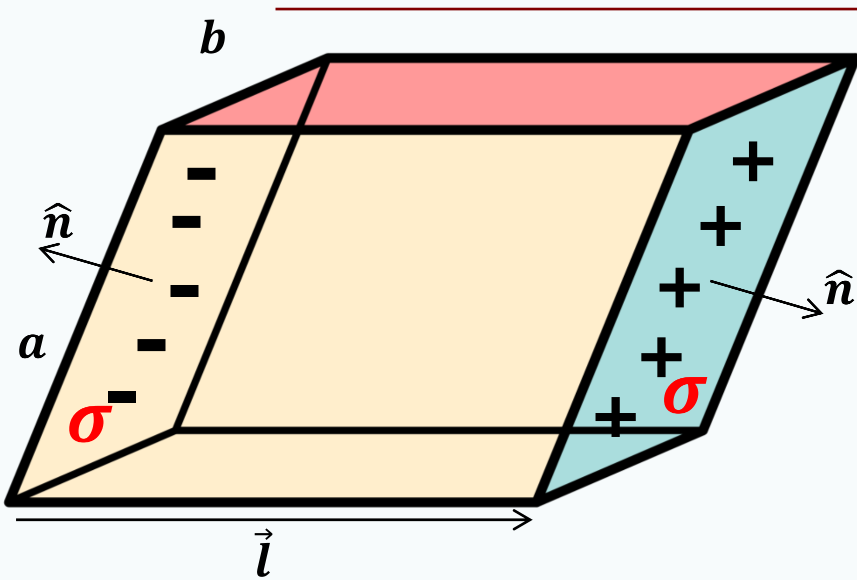
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \kappa \vec{E} = \varepsilon \vec{E}$$

- $\vec{D}$: displacement electric field

$$u_E = \frac{1}{2} \vec{E} \cdot \vec{D}$$

- energy density within dielectric

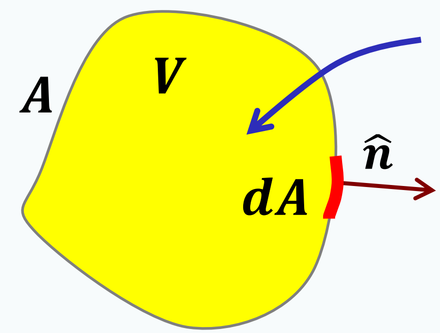
$\vec{E}$



$$\vec{P} = \frac{A\sigma\vec{l}}{V}$$

$$V = A(\vec{l} \cdot \hat{n})$$

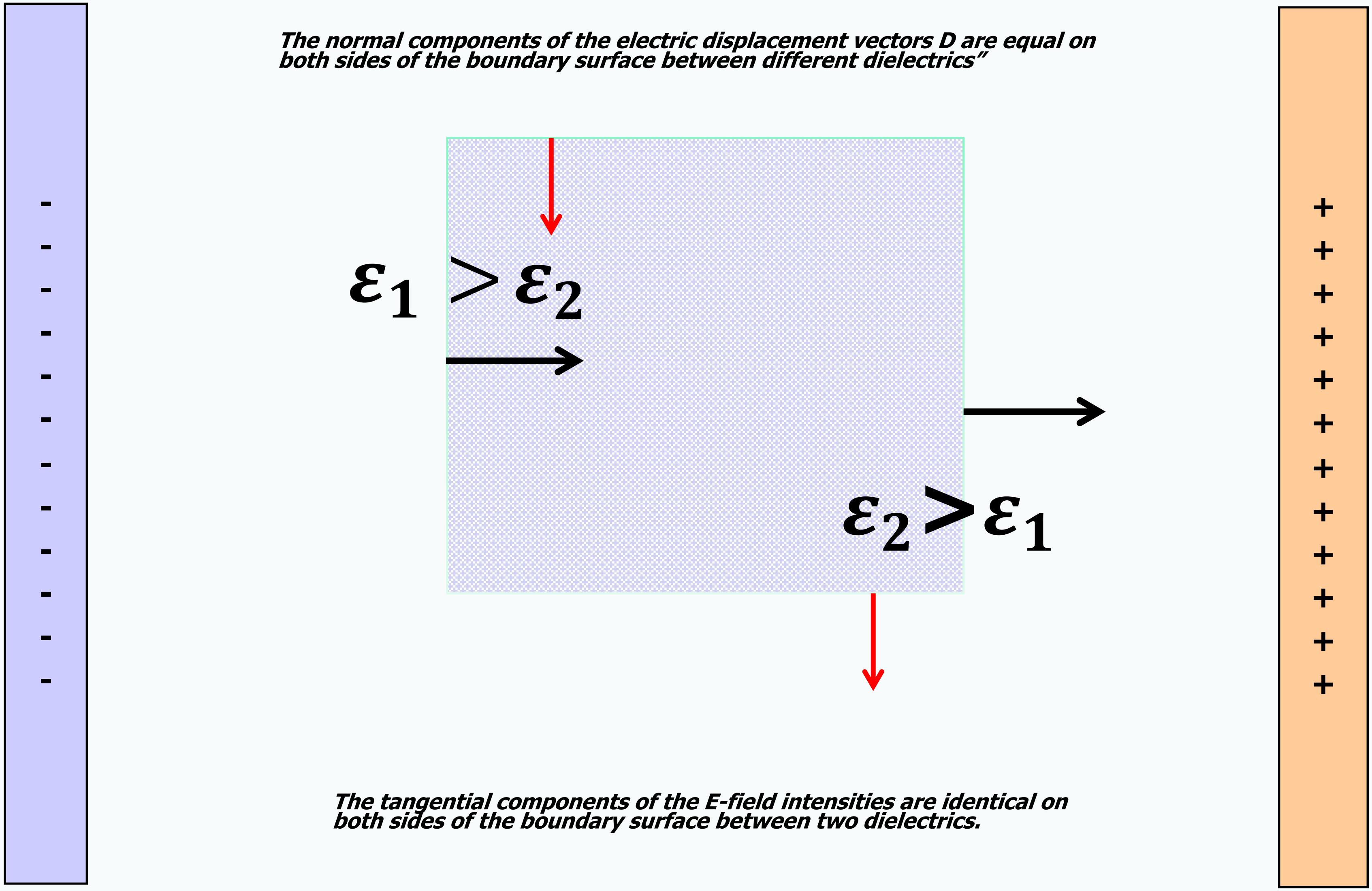
$$\sigma = \vec{P} \cdot \hat{n}$$



$$q_{ind} = -\oint \vec{P} \cdot d\vec{A}$$

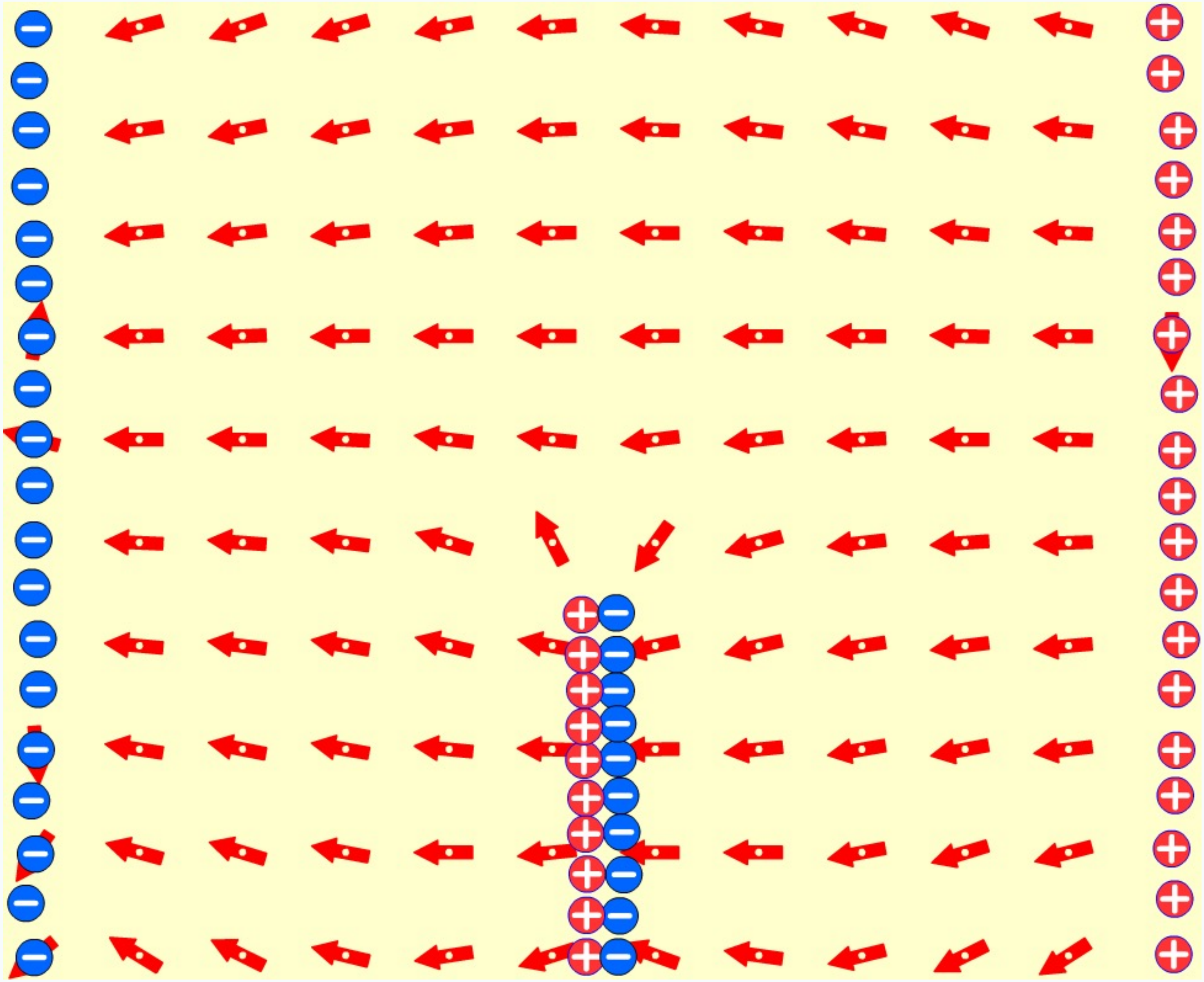
Forces on dielectrics

The normal components of the electric displacement vectors D are equal on both sides of the boundary surface between different dielectrics"



The tangential components of the E-field intensities are identical on both sides of the boundary surface between two dielectrics.

Dielectric slab in a capacitor



To remember!

- **Electrostatic potential energy of a system of charges is the work needed to bring all particles together from an infinite separation.**
- **A capacitor is a device for storing charge and energy.**
- **It is defined as charge per potential.**
- **For parallel connection of capacitors, the potential difference is the same for all capacitors.**
- **For serial connection of capacitors, the potential differences for each capacitor are added.**
- **A non-conducting material is called dielectric.**
- **Dielectrics weaken the electric field and, therefore, increase capacitance by factor κ, which is dielectric constant.**