Experimental Physics EP2

Electricity and Wave Optics

– Continuous charges –
Electric field, Gauss's law

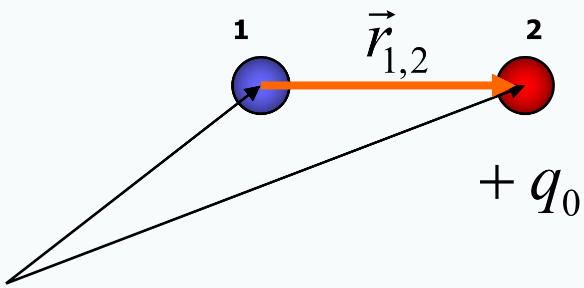
https://bloch.physgeo.uni-leipzig.de/amr/

Electric field

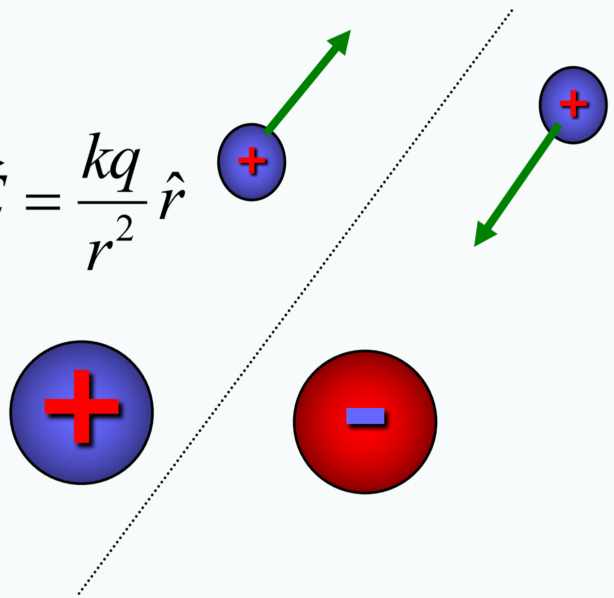
$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

$$\vec{E} \equiv \frac{\vec{F}}{q_0} \qquad \left[\frac{N}{C}\right]$$

TABLE 23.2 Typical Electric Field Values	
Source	***E* (N/C)**
Fluorescent lighting tube	10
Atmosphere (fair weather)	100
Balloon rubbed on hair	1 000
Atmosphere (under thundercloud)	10 000
Photocopier	100 000
Spark in air	$>$ 3 000 000
Near electron in hydrogen atom	5×10^{11}



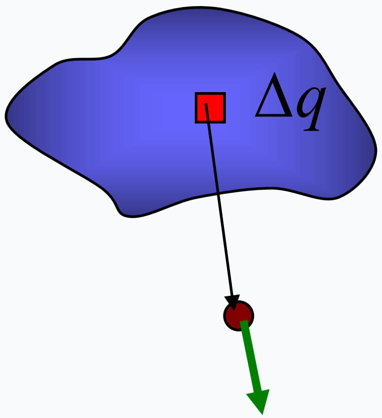
$$\vec{E} = \frac{kq}{r^2}\hat{r}$$



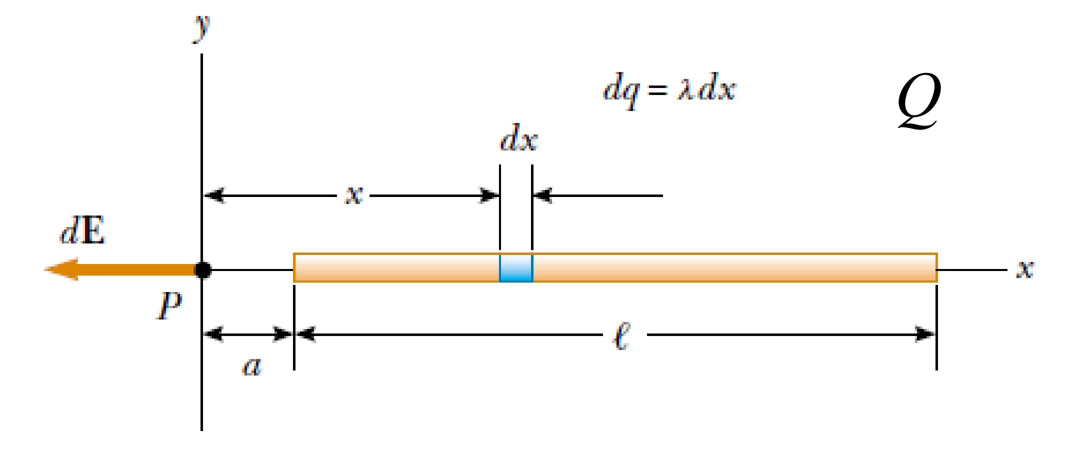
At any point the total electric field is equal to vector sum of the electric fields due to the individual charges.

$$\vec{E} = \sum_i \frac{kq_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r}$$



Selected examples



$$dE = k\frac{dq}{x^2}$$

$$dE = k\frac{\lambda dx}{x^2}$$

$$E = k\lambda\int_a^{l+a}\frac{dx}{x^2} = -k\lambda\left(\frac{1}{l+a} - \frac{1}{a}\right)$$

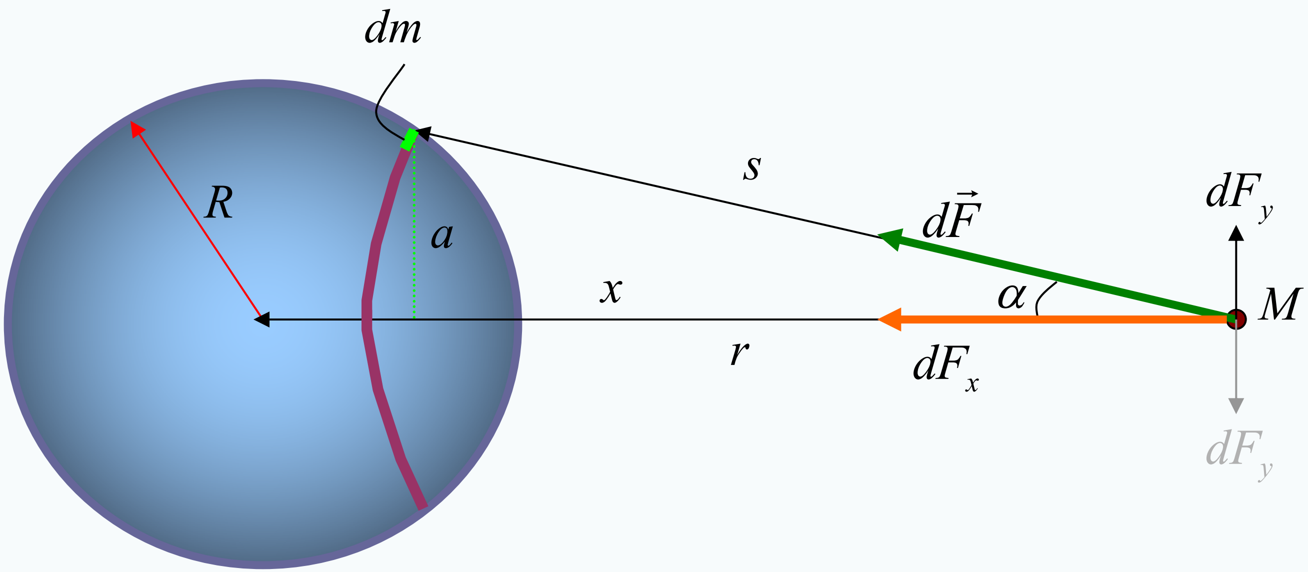
$$E = \frac{kQ}{a(l+a)}$$

$$a >> l$$

$$E = \frac{kQ}{a^2}$$

- the rod can be considered as a point-like charge

Interaction with ring



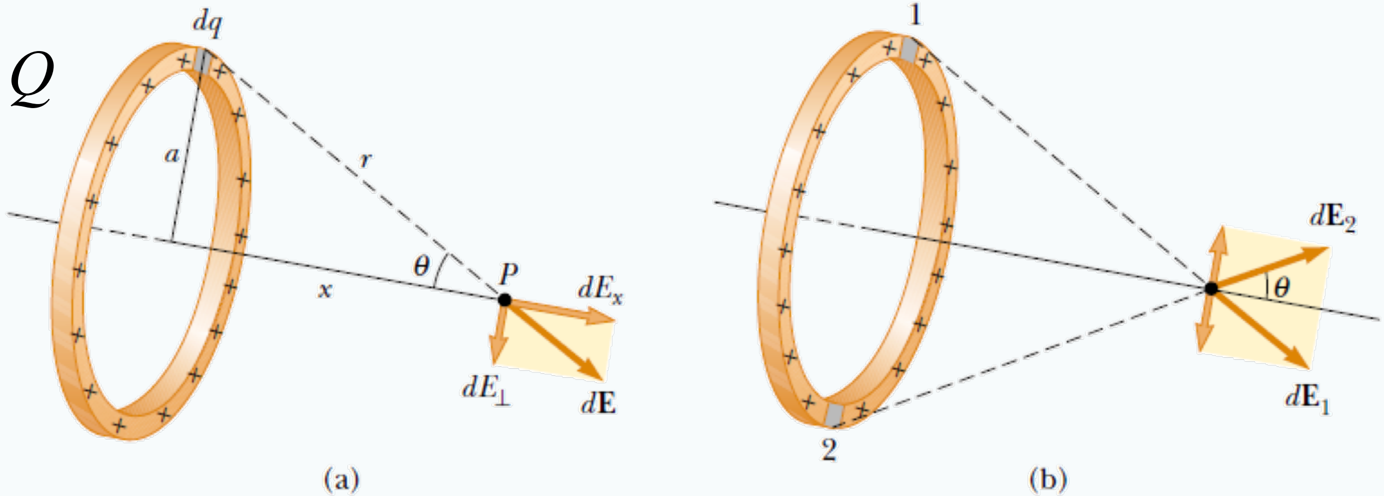
$$dF = \frac{GM}{s^2} dm \qquad dF_x = -\frac{GM}{s^2} dm \cos\alpha$$

$$F_x = -\int \frac{GM \cos\alpha}{s^2} dm = \frac{GMm_{\text{ring}}}{s^2} \cos\alpha \qquad s^2 = a^2 + x^2 \qquad \cos\alpha = \frac{x}{s}$$

integration over the ring

$$F_x = \frac{GMm_{\text{ring}}}{s^2} \frac{x}{s} = \frac{GMm_{\text{ring}} x}{\left(a^2 + x^2\right)^{3/2}}$$

Selected examples

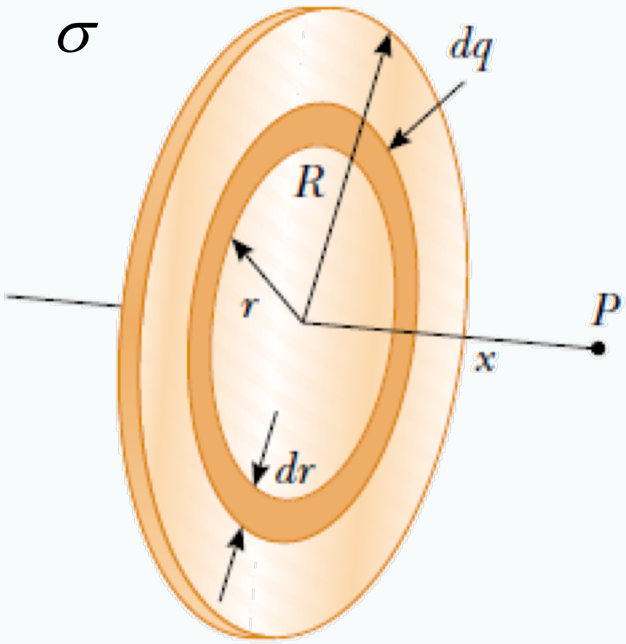


$$dE = k\frac{dq}{r^2} \qquad dE_x = dE\cos\theta \qquad \cos\theta = \frac{x}{r}$$

$$E_x = k\int\frac{dq}{r^2}\frac{x}{r} = k\int\frac{xdq}{\left(x^2+a^2\right)^{3/2}} \qquad E_x = \frac{kxQ}{\left(x^2+a^2\right)^{3/2}}$$

Selected examples

σ



$$dE_x = \frac{kx}{\left(r^2 + x^2\right)^{3/2}} \times 2\pi r \sigma dr$$

$$E_x = \pi k x \sigma \int_0^R \frac{d(r^2 + x^2)}{\left(r^2 + x^2\right)^{3/2}}$$

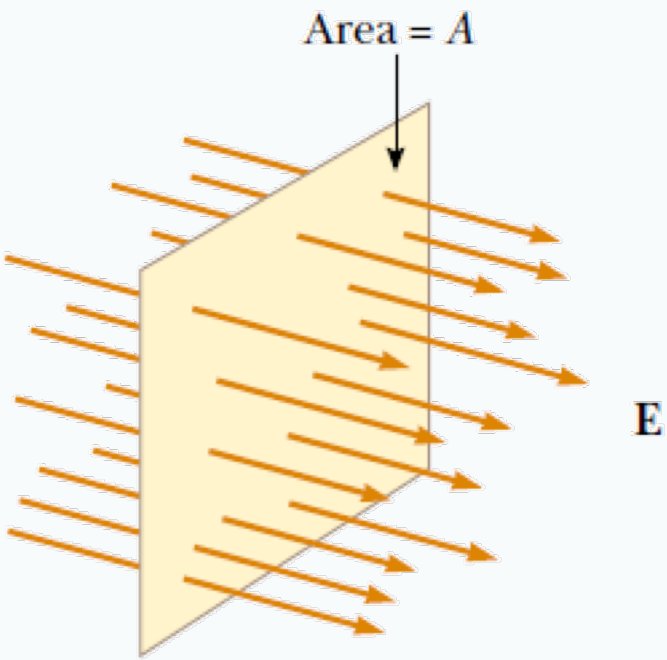
$$E_x = -2\pi k x \sigma \left(r^2 + x^2\right)^{-1/2} \Big|_0^R$$

$$E_x = 2\pi k \sigma \left(\frac{x}{|x|} - \frac{x}{\sqrt{x^2 + R^2}} \right)$$

$R >> x$

$$E_x = 2\pi k \sigma$$

Electric flux

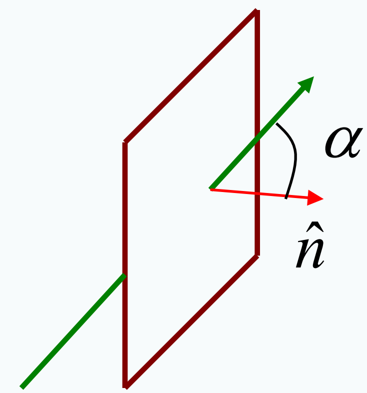


$$\Phi_E = EA$$

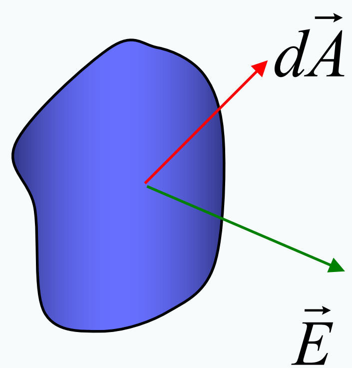
$$\left[\frac{N \cdot m^2}{C}\right]$$

Electric flux is equal to number of electric lines passing through a given surface area.

$$\Phi_E = EA\cos\alpha$$



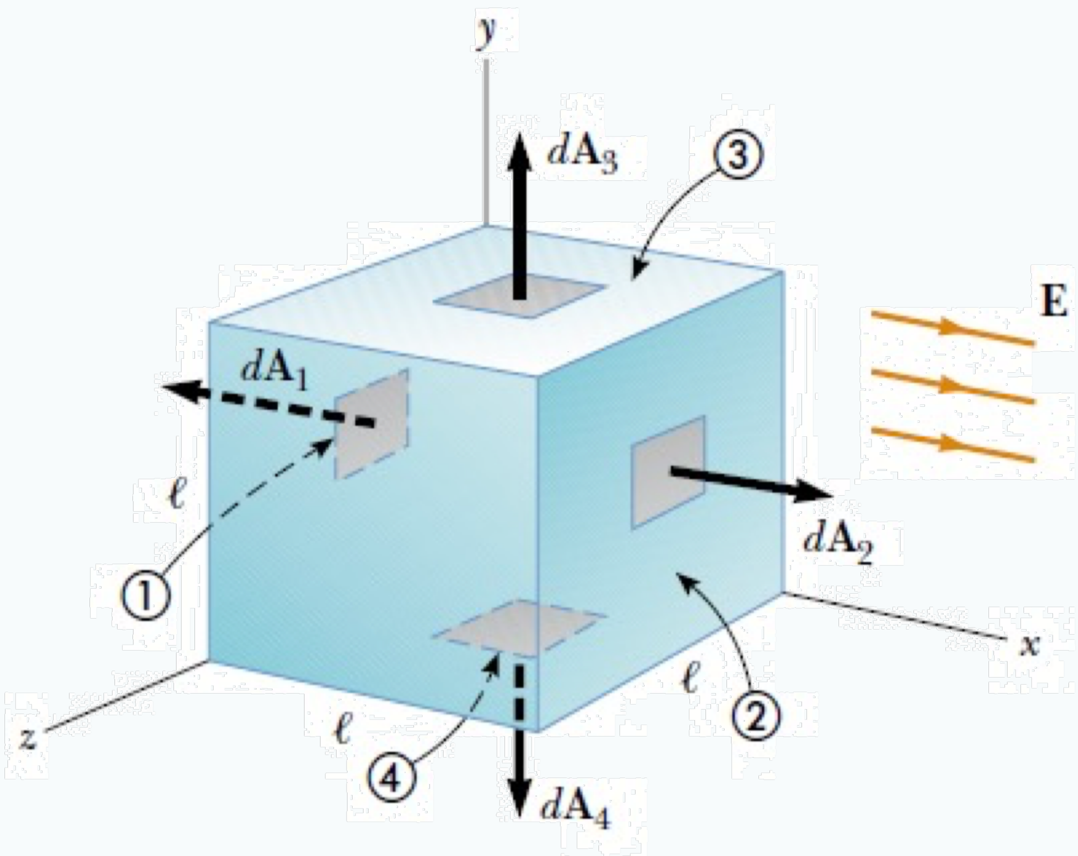
$$\Phi_E = \int_{surface} \vec{E}d\vec{A}$$



Electric flux

$$\Phi_E = \oint \vec{E} d\vec{A} = \oint E_n dA$$

- net flux through a closed surface

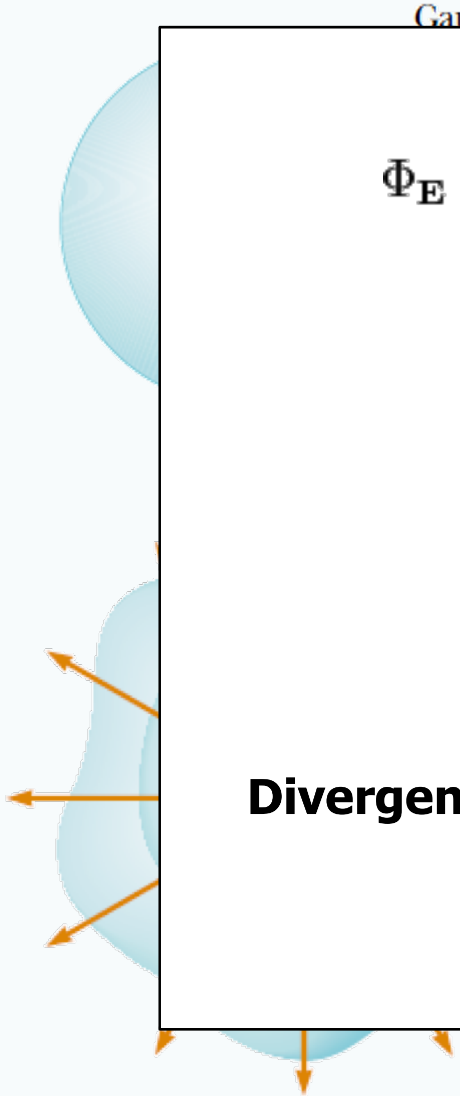


$$\Phi_1 = -El^2$$

$$\Phi_2 = El^2$$

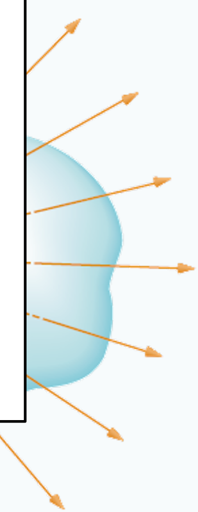
$$\Phi_{net} = 0$$

Gauss's law

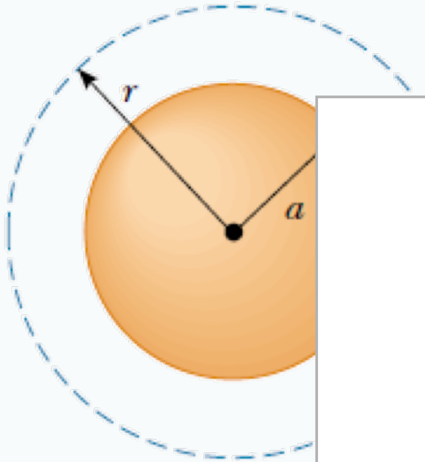


$$\Phi_{\mathbf{E}} \equiv \oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{S}$$

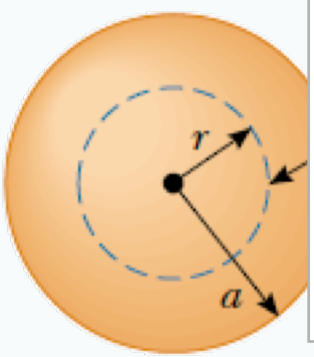
Divergence theorem $$\iiint_V \operatorname{div} \mathbf{F}\, dV = \oiint_S \mathbf{F} \cdot \mathbf{n}\, dS,$$



Application of the Gauss's law: Insulators

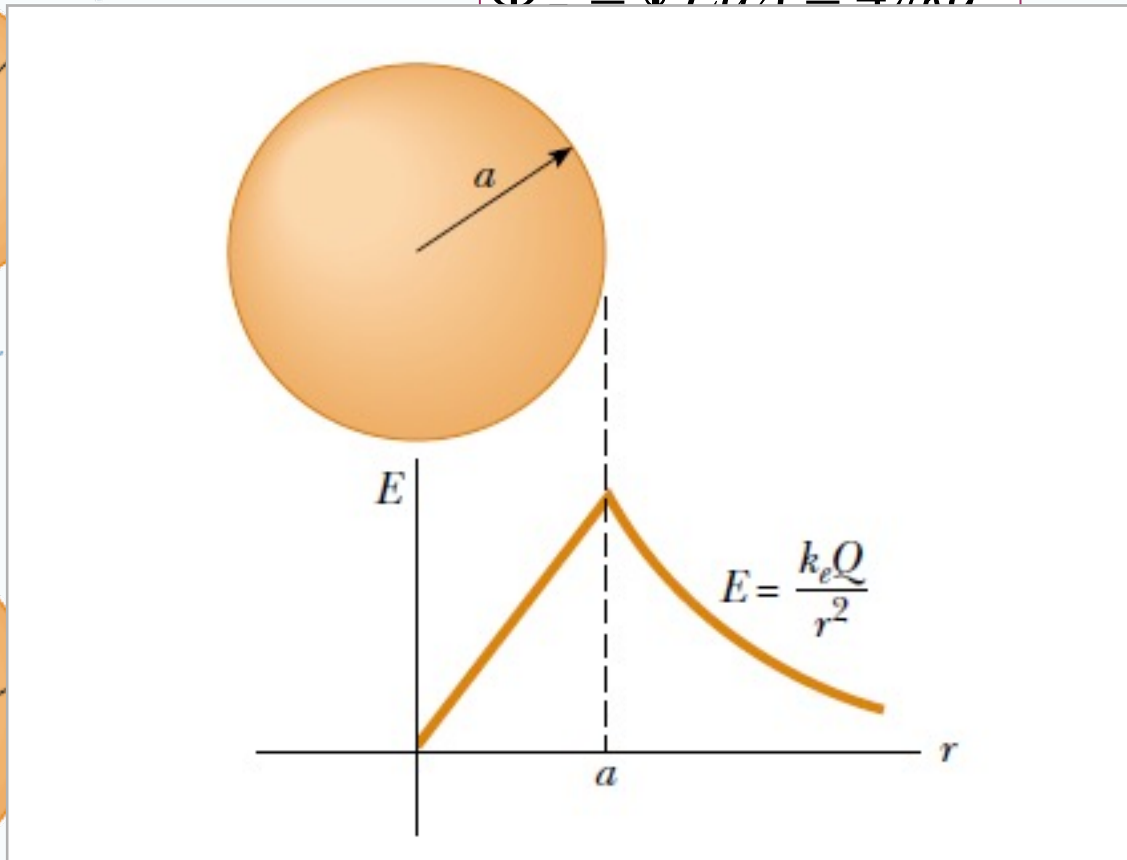


(a)



(b)

$$\Phi = \oint \vec{E}d\vec{A} = 4\pi kq$$



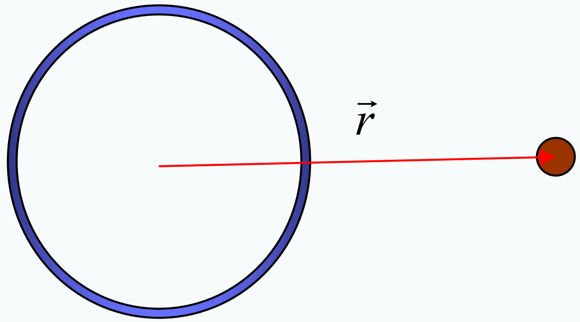
$$4\pi kQ$$

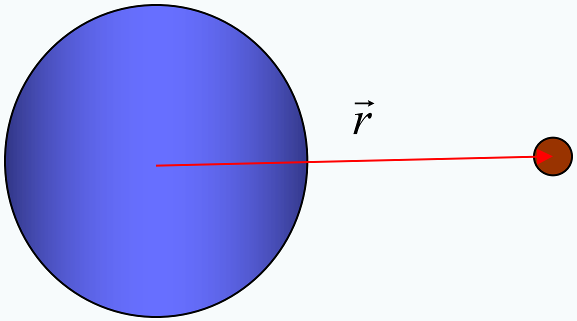
$$E = \frac{kQ}{r^2}$$

$$Q_{in} = Q\frac{r^3}{a^3}$$

$$E = \frac{kQ}{a^3}r$$

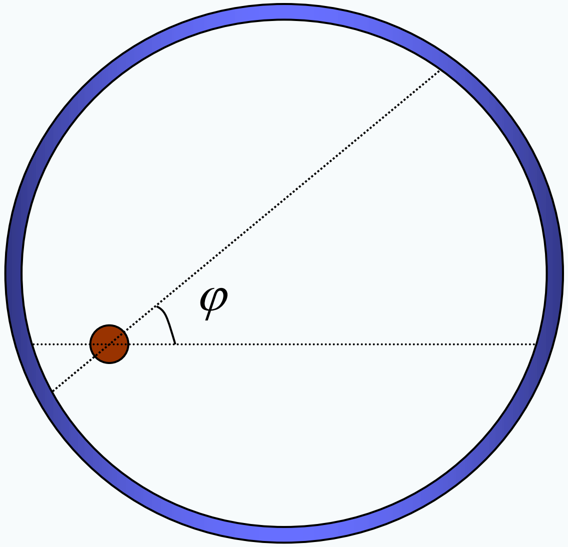
Gravitational force: extended objects

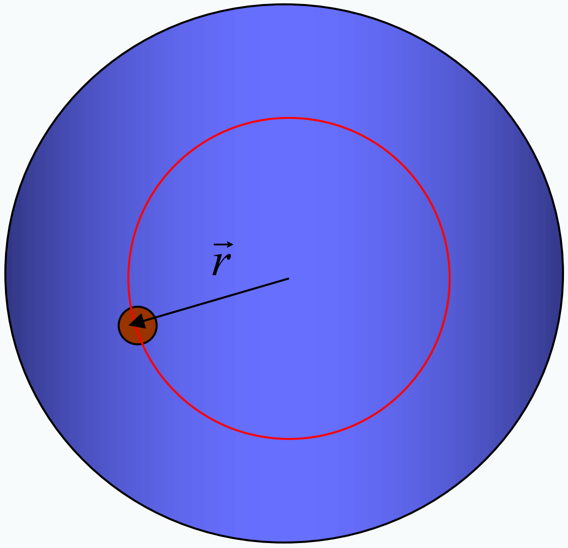




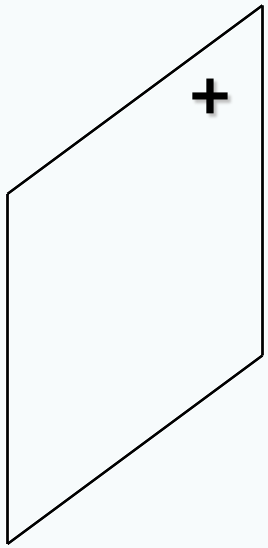
$$\begin{cases} \vec{F}_g = -\dfrac{GmM}{r^2}\,\hat{\mathrm{r}} & \mathrm{r} > \mathrm{R} \\ \vec{F}_g = 0 & \mathrm{r} < \mathrm{R} \end{cases}$$

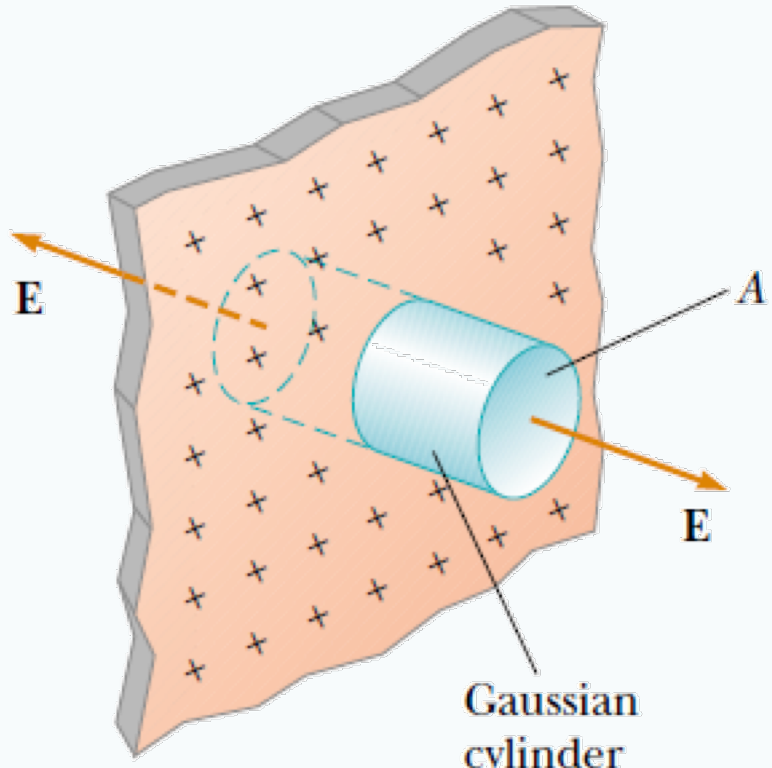
$$\begin{cases} \vec{F}_g = -\dfrac{GmM}{r^2}\,\hat{\mathrm{r}} & \mathrm{r} > \mathrm{R} \\ \vec{F}_g = -\dfrac{GmMr}{R^3}\,\hat{\mathrm{r}} & \mathrm{r} < \mathrm{R} \end{cases}$$





Application of the Gauss's law: Insulators



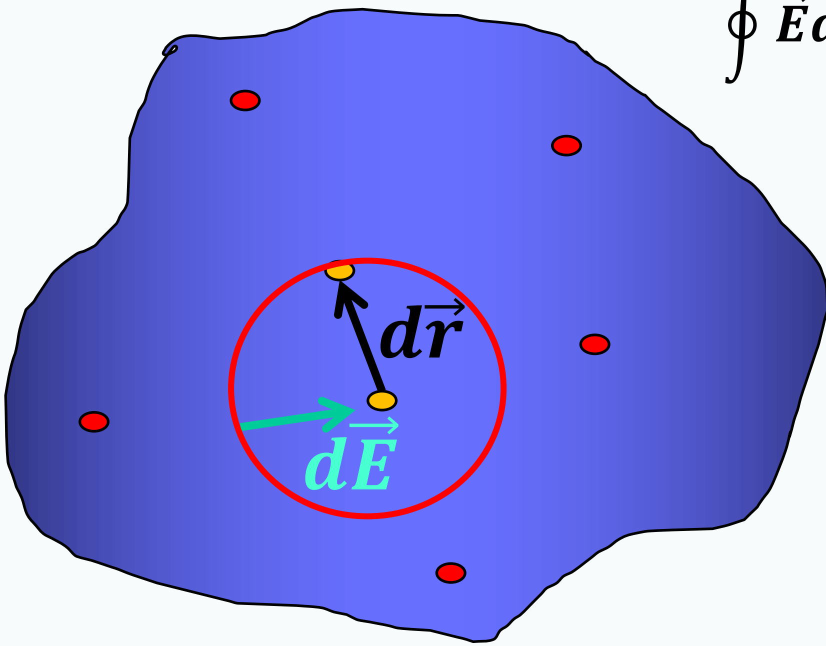


$$\Phi_E = 2EA$$

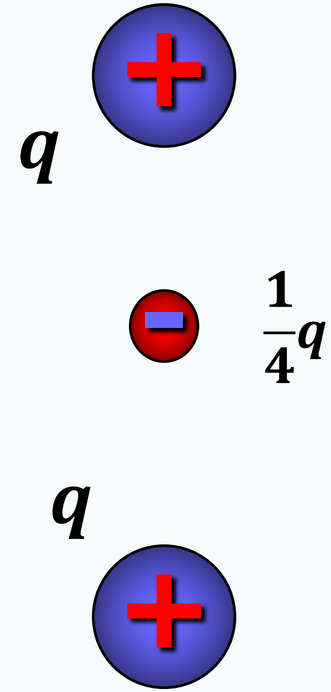
$$\Phi_E = 4\pi k A \sigma$$

$$E = 2\pi k \sigma$$

Earnshaw's theorem



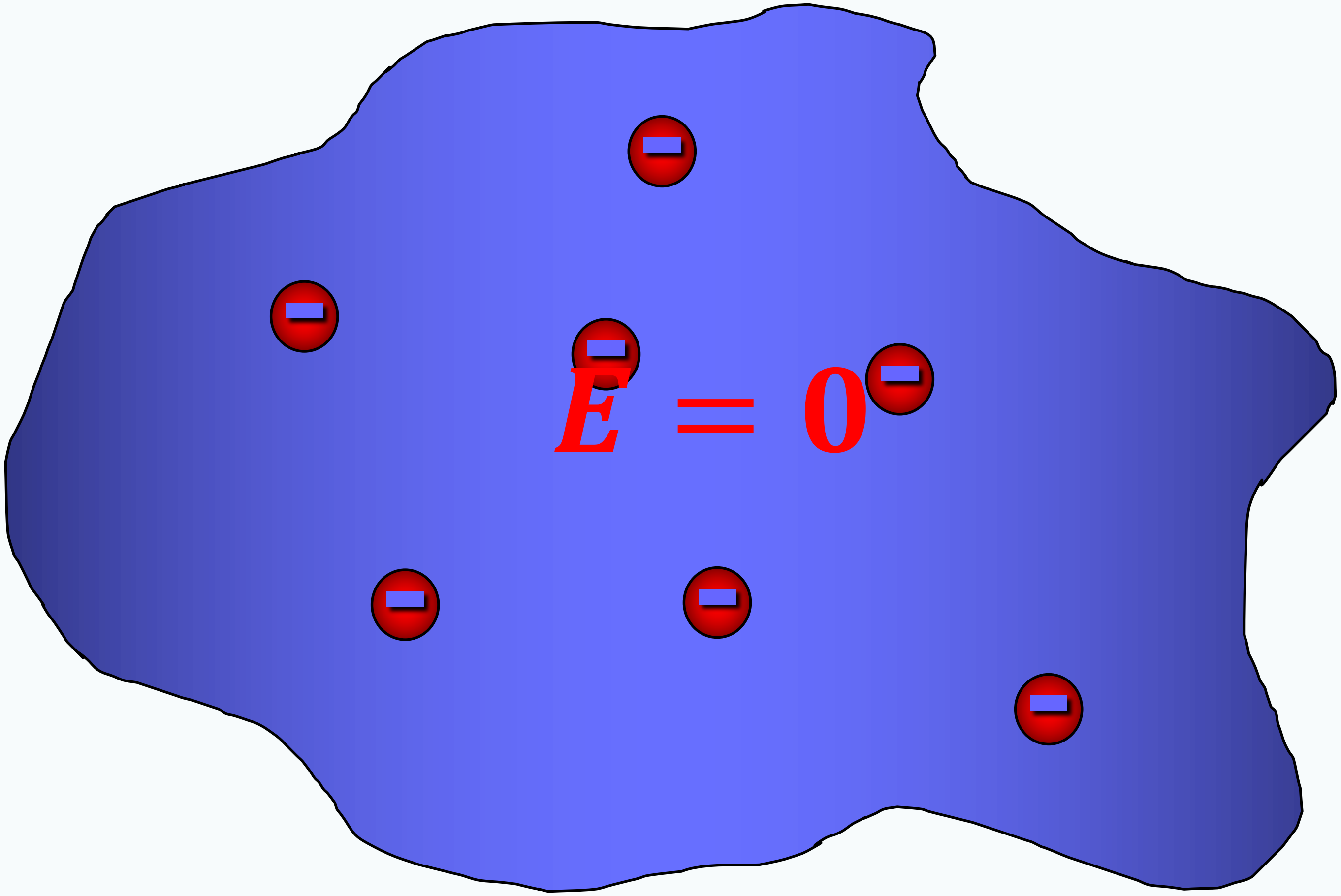
$$\oint \vec{E} d\vec{A} \neq 0$$



Any equilibrium configuration of point-like charges interacting solely via electrostatic interactions is unstable.

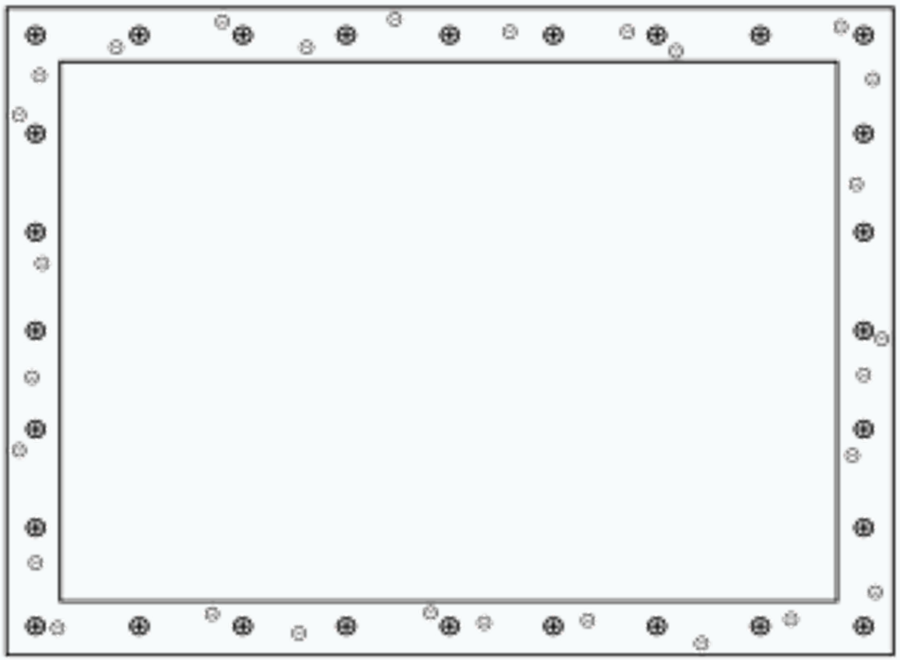
The theorem is applicable for any inverse-square law forces, including gravitational and magnetic.

Earnshaw's theorem



Electric Field Lines

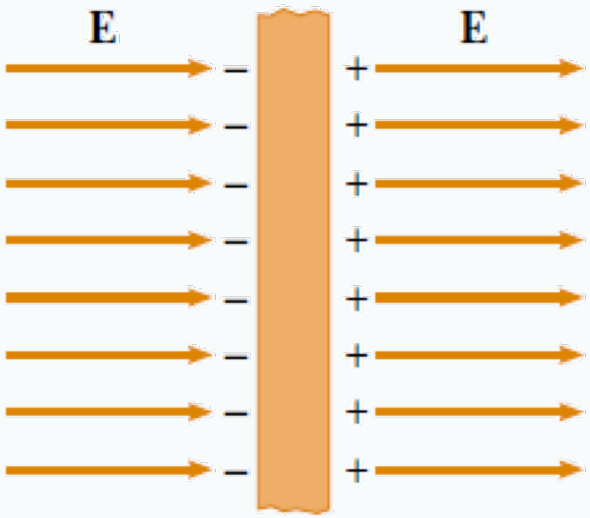
shielding the electric field – the Faraday cage

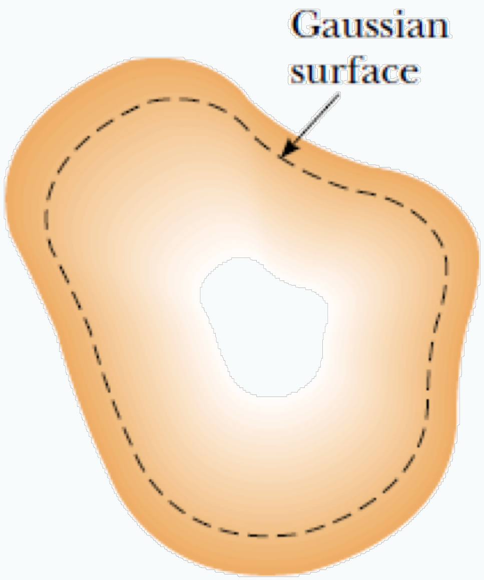


Application of the Gauss's law: Conductors

Electrostatic equilibrium:

- **Electric field within a conducting body is zero everywhere.**
- **Electric charges within a conductor are locates on its surface.**
- **Electric field outside a conductor is perpendicular to its surface and is $4\pi k\sigma$.**
- **If surface is irregular, then electric field is greatest where the curvature is highest.**





To remember!

- At any point the total electric field is equal to the vector sum of electric fields due to individual charges.
- Electric flux is equal to number of electric field lines penetrating given surface - the scalar product of electric field and the area element (pointing along the normal to the surface).
- The net flux through any closed surface enclosing charge q is equal to $4\pi kq$.
- The net flux through any closed surface enclosing no charge is zero.
- At electrostatic equilibrium there is no any macroscopic charge transport.
- Electric field within a conductor is zero, all charge is located at the surface.